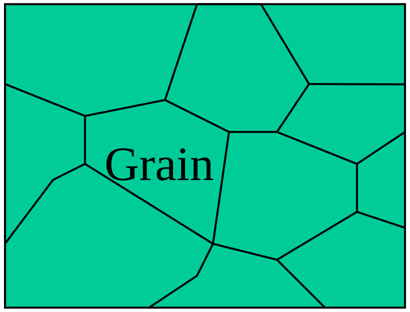
Scaling (II)

Scaling in material properties

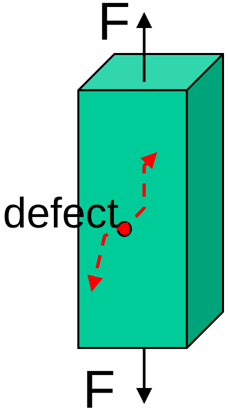
(Elastic modulus, Poisson's ratio, fracture strength, yield stregnth, residual in-plane stress, conductivity, ..)

The properties of thin-film material are often different from their bulk / macroscopic form. The disparity arises from

- The different fabrication processes
  - Macroscale: casting, machining, milling, cutting, 3D printing
  - Microscale: deposition (single-crystal,amorphous) , etching,..

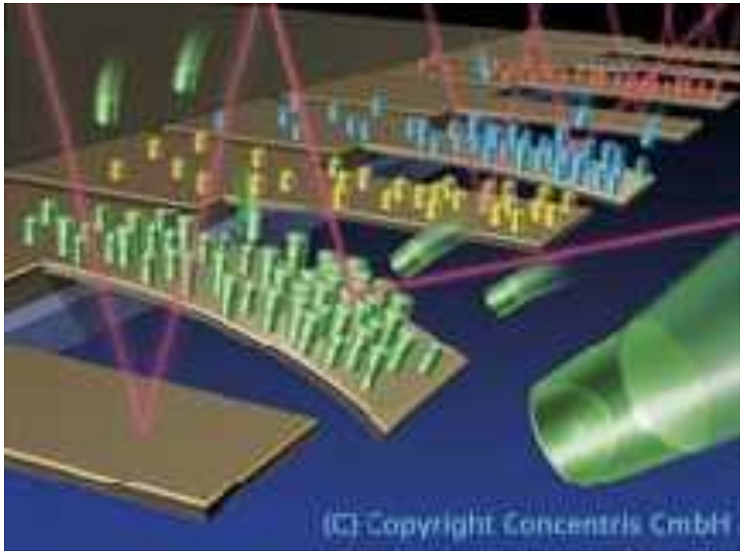


- The assumption of homogeneity
  - Macroscale: usually is accurate
  - Microscale: unreliable when used to model devices that have dimensions on the scale of individual grains
- The defect density
  - Macroscale: high defect counts
  - Microscale: low defect counts

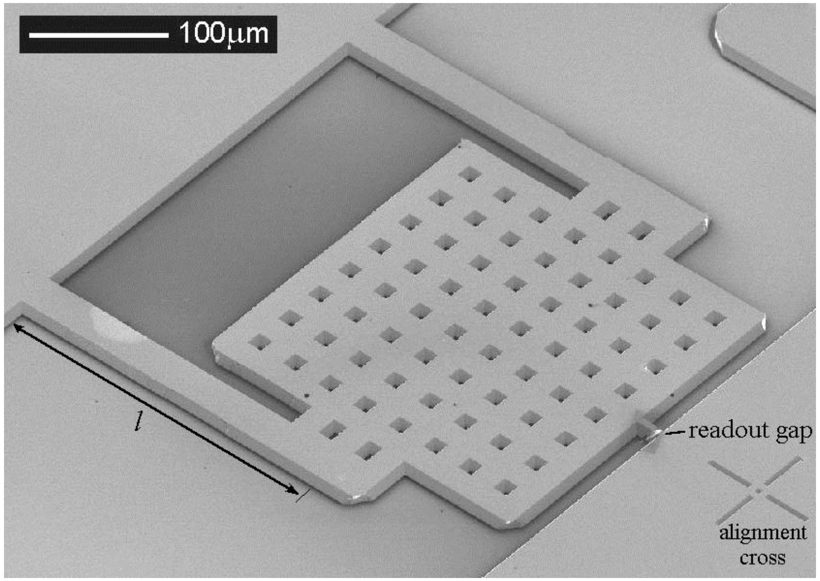


$$\# \text{ defects} = D_{defect} * \text{Volume}$$

Scaling in a mechanical system (e.g. a cantilever beam)

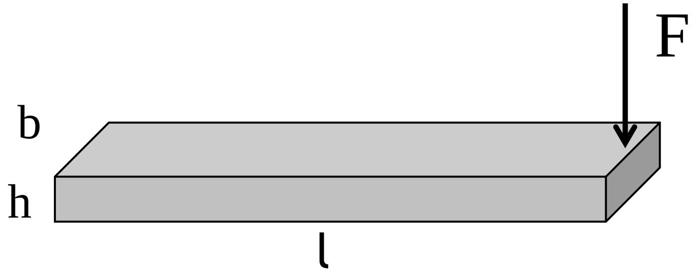


Cantilever biosensor



Cantilever accelerometer

- Stiffness of a cantilever



$$k \approx \frac{\text{force}}{\text{deflection from equilibrium}}$$

$$k = \frac{EI}{l^3}$$

I: area moment of inertia around the end of the beam

$$k \text{ » } E\frac{b \times h^3}{l^3} \qquad \sim L^1$$

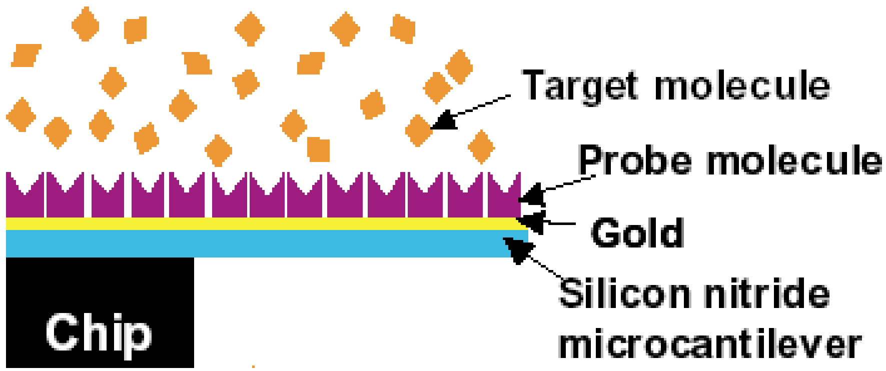
E is the elastic modulus of the spring material

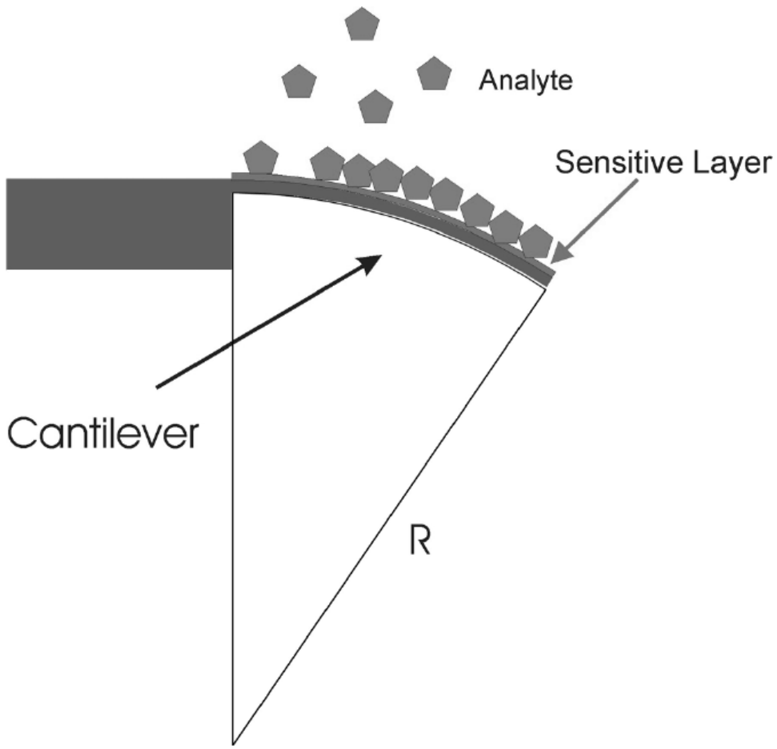
Implications ?

Displacement x = F / k ;
F = m·a ~ L^3
x ~ L^2 (a beam of 10-fold smaller in size has 100-fold smaller deflection under a same acceleration)

- (+) A micro accelerometer can experience more than 100,000 g
- (-) for the devices requiring proof mass (e.g. accelerometer) must have a highly sensitive motion-detection signal-transduction system

- Cantilever as biosensors





In **<u>static mode detection</u>**, the deflection of individual cantilever depends on the stress induced by binding reaction of the specific compounds to the interface. Typically, the cantilever surface is coated with a metallic layer (Au) and subsequently activated by binding a receptor molecule directly via a thiol group to the interface.

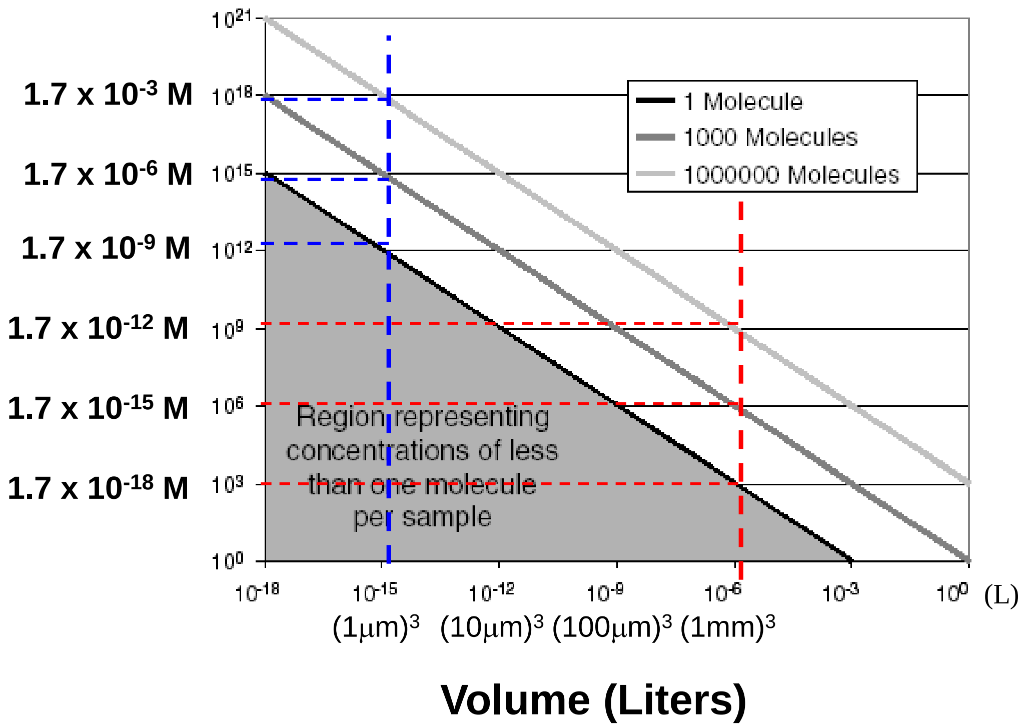
Stoney's Law

$$\sigma = Et^2[6\mathrm{R}(1-\gamma)]^{-1}$$

σ: stress
R: Radius of the curvature
E: Young's modulus
γ : Poisson ratio
T: thickness of the cantilever

$$R = \frac{Et^2}{6\sigma(1-\gamma)} \quad \sim \mathrm{t}^2$$

Scaling in a chem/biological system



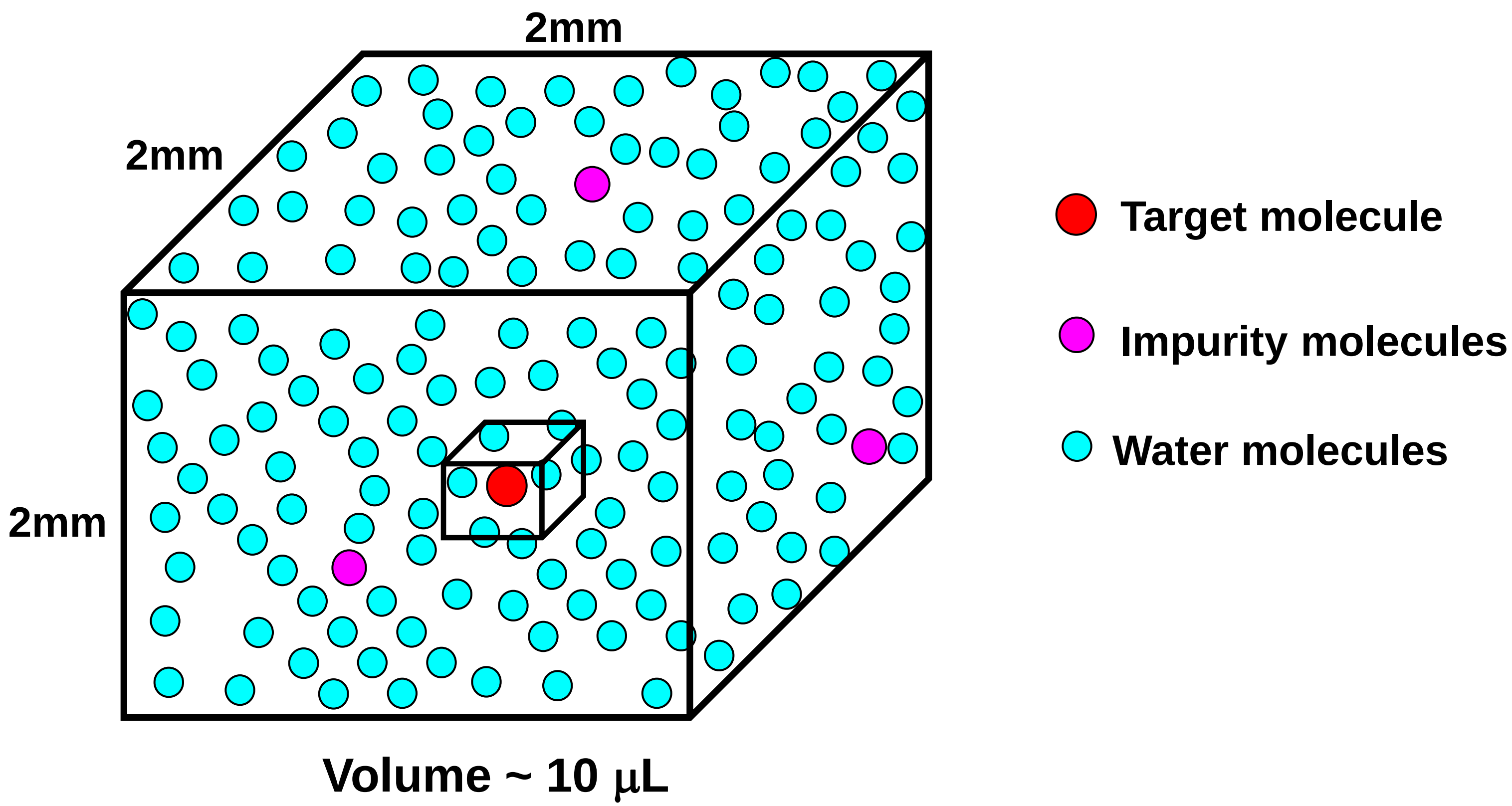
- Volume
- Concentration
- Sensitivity

Volume	1µl	1nL	1pL	1fL	1aL
# of molecules in 1 µM solution	6×10^{11}	6×10^{8}	6×10^{5}	600	0.6

- Low volume requires a sensing method with higher sensitivity
- Measurement with extremely low volume enables single-molecule measurement at the physiological concentration level

Scaling in a chem/biological system (cont.)

- Noise Reduction by Downscaling Detection Volume



10^{20} excess background molecules over 1 target molecule

Scaling in a thermal system

- Thermal mass (thermal capacity)

$$M \cdot C_p \sim L^3$$

C_p : specific heat

- Heat transfer
  - Heat conduction rate $Q = -kA\dfrac{\Delta T}{\Delta x}$ $\sim L^1$

k: thermal conductivity
A: area
T: temperature
x: distance

  - Heat convention rate $Q = qA = hA\Delta T$ $\sim L^2$

h: heat transfer coefficient

Thermal capacity will scale down more rapidly than heat transfer

- A more careful analysis is required to predict the thermal behavior of miniature structure when they are scaled down to sub-micro dimensions. The dimensions are of the same scale as the quantum mechanical ***phonon***, or ***quantum of lattice vibration***

Scaling in surface tension

- Surface tension force $= \gamma \cdot L \sim L^1$

Example : Surface tension ($\gamma_{water} \sim 72$ mN/m)

- A bug (10 mg) needs 1 mm of foot edge to walk on water
- A human (60 kg) would need feet with 8000 m to walk on water

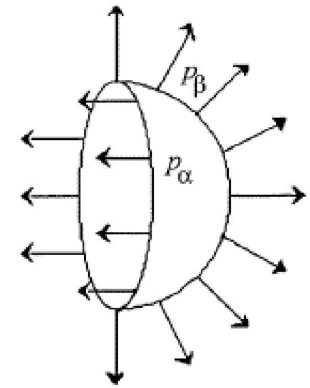
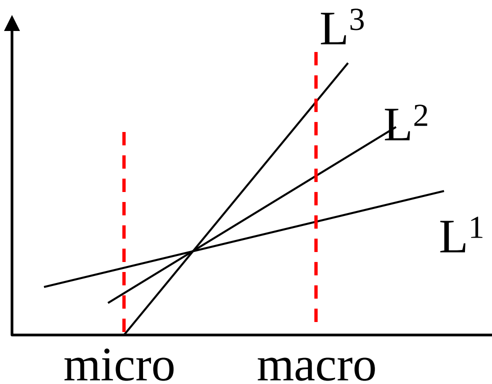


Figure 1. Imaginary hemispherical section of a spherical liquid drop. The arrows pointing radially outwards represent forces due to the pressure difference ($p_\alpha - p_\beta$). The arrows pointing to the left represent forces due to surface tension.

$$\Delta P = \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \sim L^{-1}$$

- 20 μm hydrophilic channel filled with water, ΔP across meniscus is 12.5 kPa

- What are the implications for micro devices ?
  - Stiction problems during releasing structures
  - Surface-tension related forces becomes effective



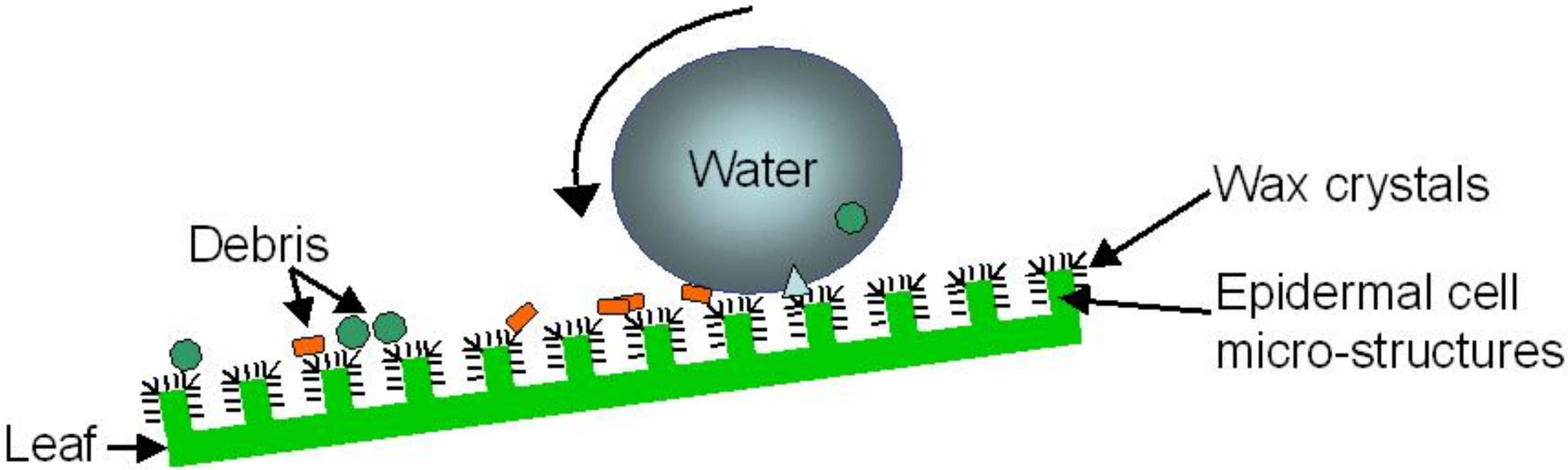
The Lotus Effect

Lotus leaves and nasturtium leaves are self-cleaning due to nano and microscale structures and a waxy coating.

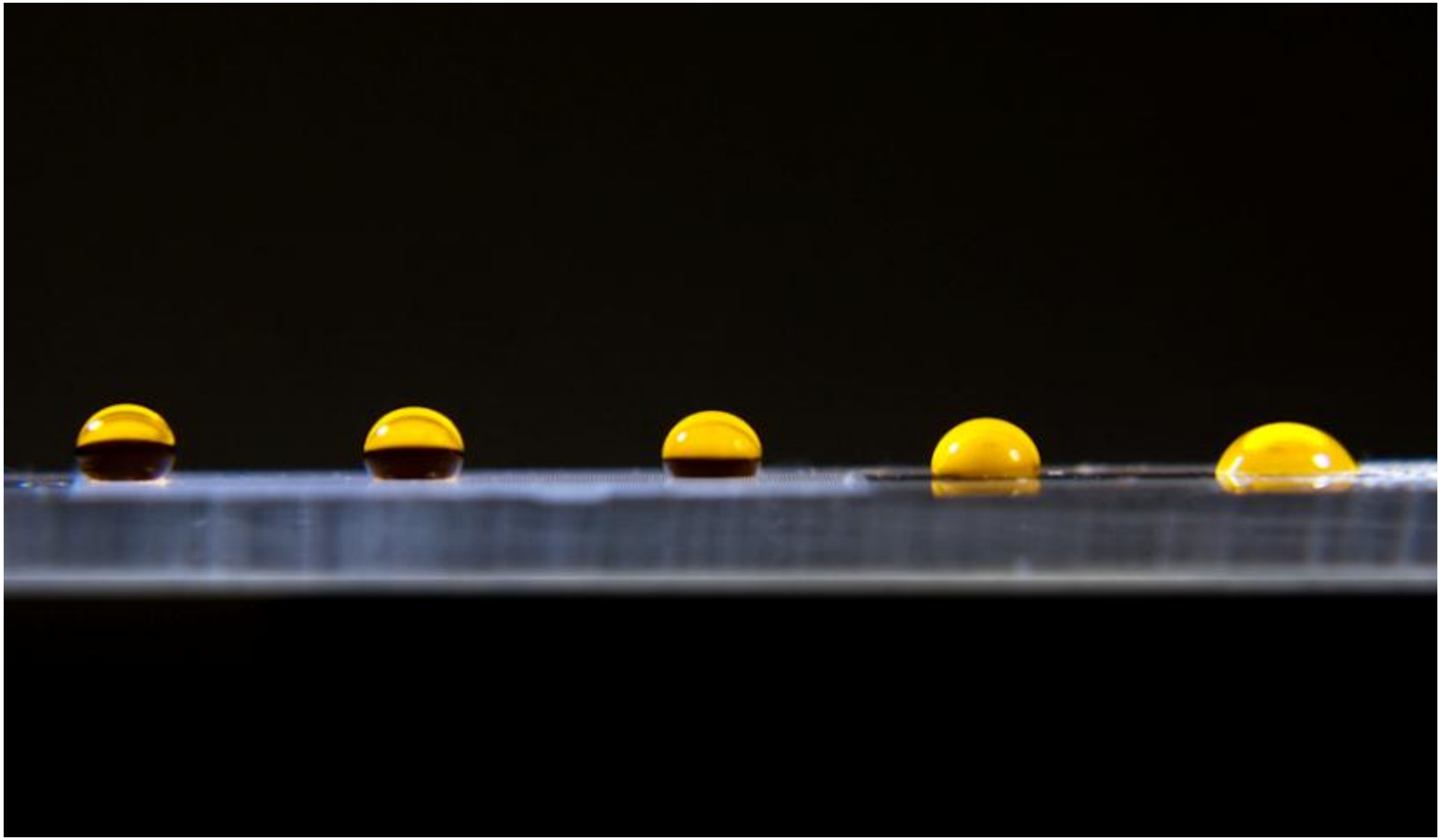
Together these features create a superhydrophobic surface.

Close-up of a nasturtium leaf, which exhibits the **Lotus Effect**, with a droplet of water

The Lotus Effect



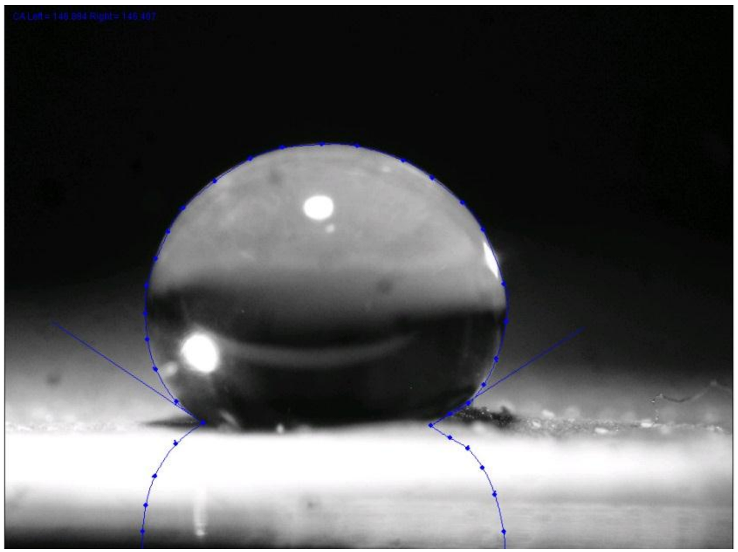
Droplet "Digital Microfluidic" Technology



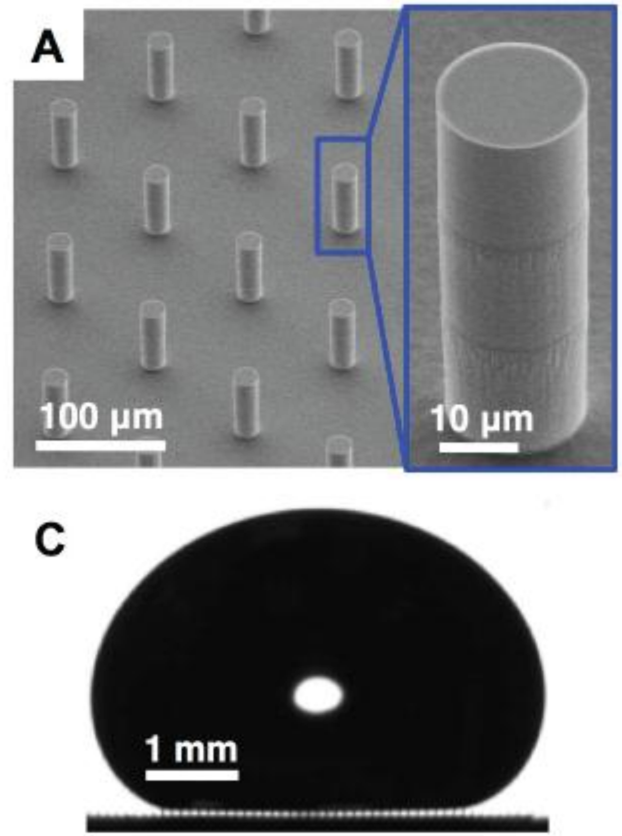
(Photo by Y Zhang & C Beh)

Grand Prize Winning Photo, 2013 JALA & JBS Art of Science Contest
Society of Laboratory Automation and Screening (slas)

Superhydropobicity



A water drop on a lotus surface showing contact angles of approximately 147° .



A water drop on an array of silicon micro posts showing contact angles of approximately 160° .

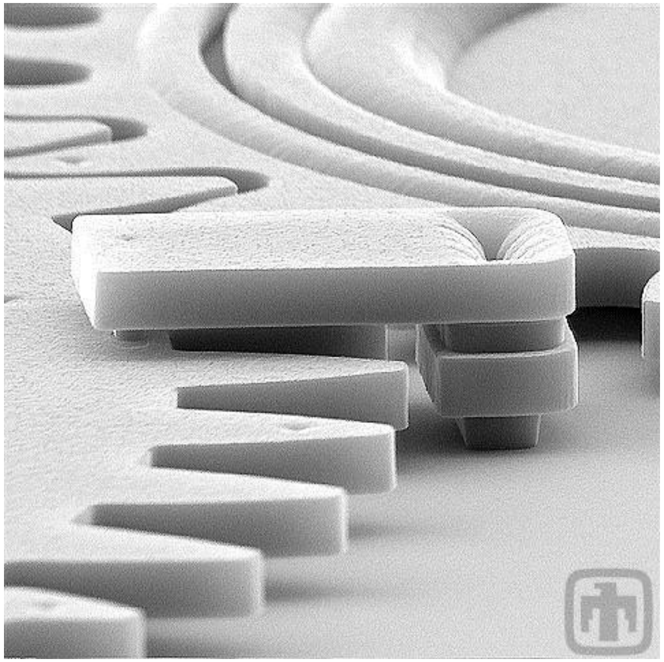
Scaling in friction

The sources of friction

- Capillary forces ~ L^1
- Adhesive surface forces such as van der Waals, hydrogen bonding, electrostatic ~ L^2

Surface roughness vs friction

- Macroscale contact occurs between few rough protrusions
- When the surface is well polished and becomes highly smooth, the contact occurs over larger area



Friction in MEMS

- To suspend structure or dimple to avoid or reduce friction

Scaling in diffusion

- Diffusion times (particle and thermal)

Mean first passage time of free diffusion $\tau = \frac{x^2}{6D}$

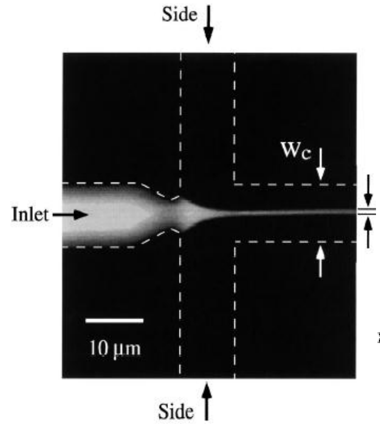
Einstein relation $D = \frac{kT}{6\pi\mu r}$

$$\boxed{t_{Diff} \sim L^2}$$

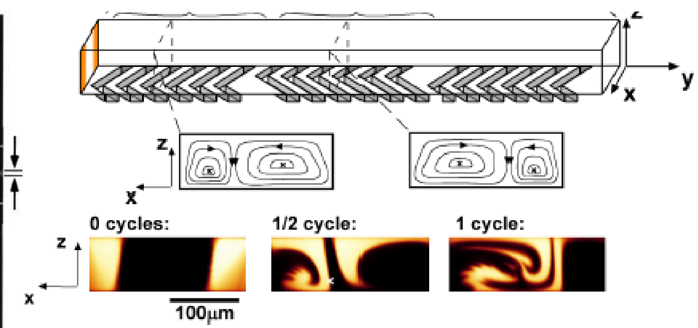
- **Diffusion time of a macromolecule with D of 10^{-5} cm²/s**

Volume	1 μL	1 nL	1 pL	1 fL	1 aL
Length of cube side *x*	1 mm	100 μm	10 μm	1 μm	100 nm
Time to diffuse *x*	500 s	5 s	0.05 s	0.5 ms	0.05 ms

- Mixing at microscale mediated mainly by diffusion



Rapid Mixing



Chaotic mixer

Scaling in a fluidic system / Dimensionless Numbers

Scaling Reynolds number

$$\mathrm{Re} = \frac{\text{Inertial force}}{\text{Viscous force}} \approx \frac{\rho L^3 \cdot dV/dt}{\mu \cdot (dV/dL) \cdot L^2} \approx \frac{\rho L^3 \cdot V/t}{\mu \cdot (V/L) \cdot L^2} \approx \frac{\rho L^3 \cdot V/(L/V)}{\mu \cdot (V/L) \cdot L^2}$$

$$\mathrm{Re} = \frac{\rho \cdot V \cdot L}{\mu} \qquad \sim L^2$$

ρ: density of the fluid
L: characteristic length
V: velocity
μ: viscosity

Laminar flow	• Bacteria in water	10^{-6}
	• Marble falling in honey	10^{-2}
	• Tropical fish	10^{2}
	• Dragonfly	10^{3}
Turbulent flow	• Car	10^{6}
	• Airplane	10^{7}
	• Whale	10^{8}

Scaling Weber number

$$\text{We} = \frac{\text{Inertial force}}{\text{Surface tension force}} \approx \frac{\rho L^3 \cdot V/(L/V)}{\sigma \cdot L}$$

$$We = \frac{\rho \cdot V^2 \cdot L}{\sigma} \qquad \sim \mathrm{L}^3$$

σ: Surface tension

Scaling Capillary number

$$\text{Ca} = \frac{\text{Viscous force}}{\text{Surface tension force}} \approx \frac{\mu \cdot (V/L) \cdot L^2}{\sigma \cdot L}$$

$$Ca = \frac{\mu \cdot V}{\sigma} \qquad \sim \mathrm{L}$$

The surface tension force dominates over the viscous force in the micro- and nanoscale.

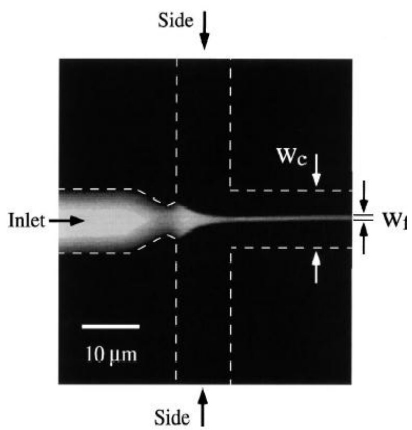
Scaling Peclet number

$$\text{Pe} = \frac{\text{Convention}}{\text{Diffusion}} = \frac{\text{Diffusion time}}{\text{Convention time}} = \frac{L^2/D}{L/V}$$

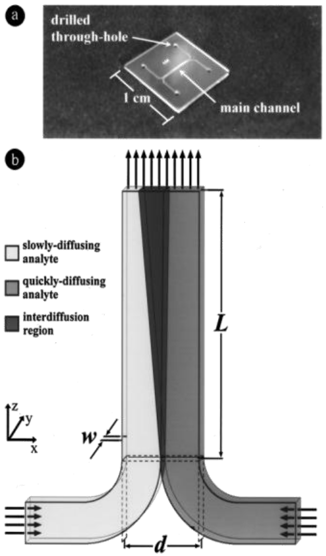
$$Pe = \frac{L \cdot V}{D} \qquad \sim L^2$$

σ: Surface tension

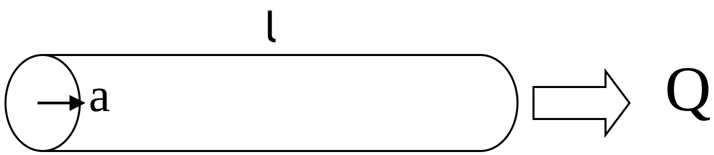
The diffusion time may approach the order of the convention time. This fact leads to the possibility of real-time measurements of reaction kinetics

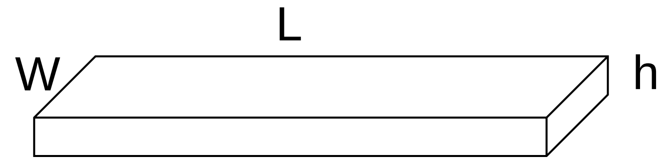


Rapid Mixing



Scaling in Pressure-pumping

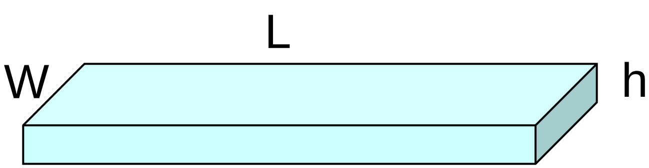




Pressure drop $\Delta P = \dfrac{8\mu l Q}{\pi a^4}$ $\sim L^{-3}$

$\Delta P = \dfrac{12\mu L}{W h^3} Q$ $\sim L^{-3}$

e.g.



$\mu_{water,\ 20\,C} = 11.3 \cdot 10^{-4}$ (Pa·s)

- **(W,h,L)= (100 μm, 30μm, 3cm), Q= 1 μl/min**

  $\Delta P = 1.5 \cdot 10^5$ (Pa) = 1.5 atm
- **(W,h,L)= (100 μm, 0.3μm, 3cm), Q= 1 μl/min**

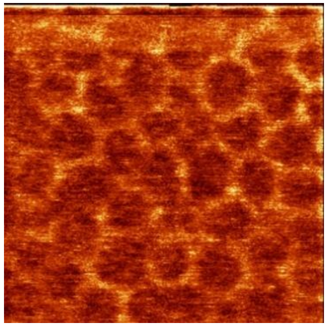
  $\boxed{\Delta P = 1.5 \cdot 10^6 \text{ atm}}$

Pressure-driven pumping becomes very difficult !!

Scaling in color

- Bulk gold appears yellow in color
- Nanosized gold appears red in color
  - The particles are so small that electrons are not free to move about as in bulk gold
  - Because this movement is restricted, the particles react differently with light

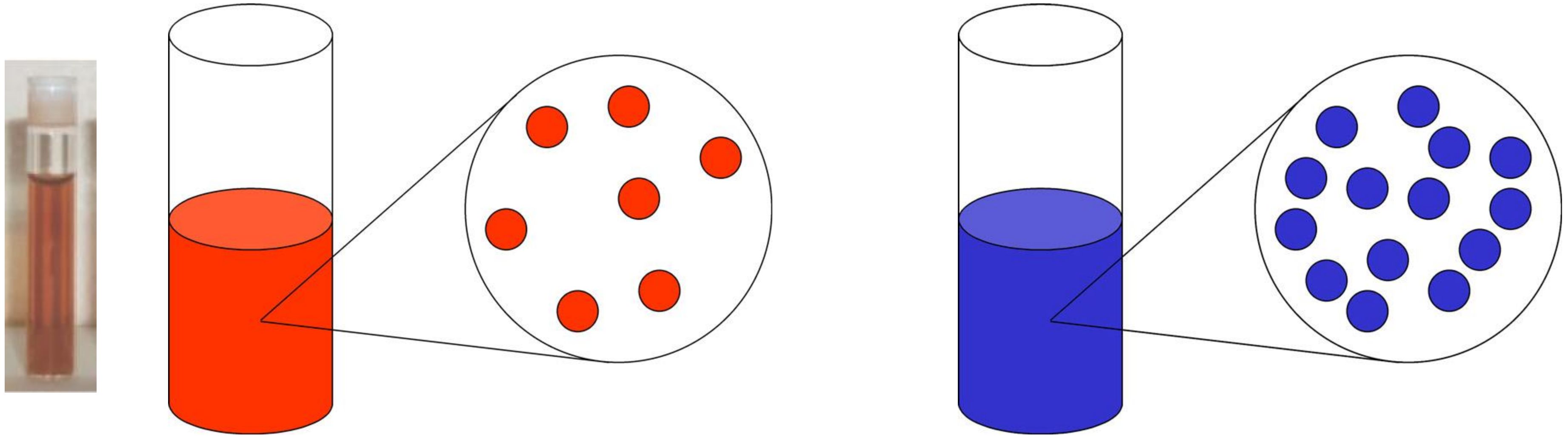
“Bulk” gold looks yellow



12 nanometer gold particles look red

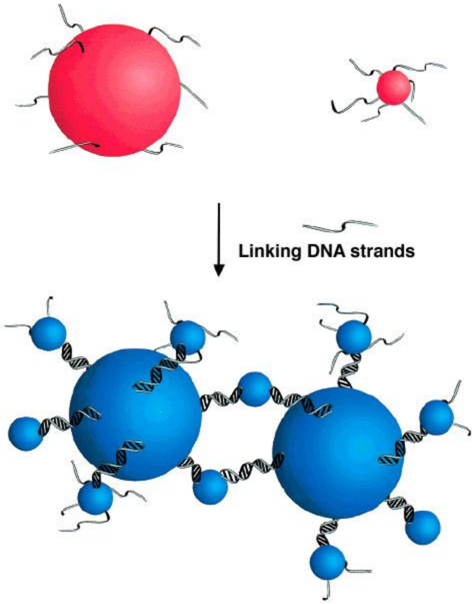
Sources: http://www.sharps-jewellers.co.uk/rings/images/bien-hccncsq5.jpg
http://www.foresight.org/Conferences/MNT7/Abstracts/Levi/

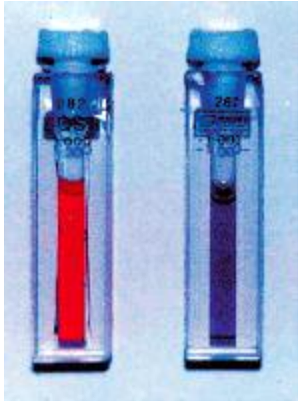
“Color change” of Au Nanoparticle

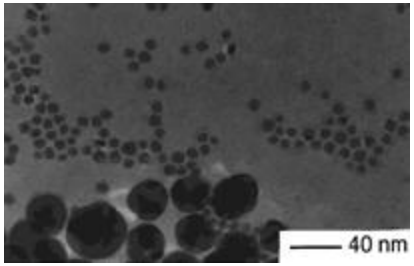


- Au nanoparticles suspend in solutions
- “Transparent”
- The color of particles comprised of 1,000-10,000 gold atoms depends on the **distance** between the particles.

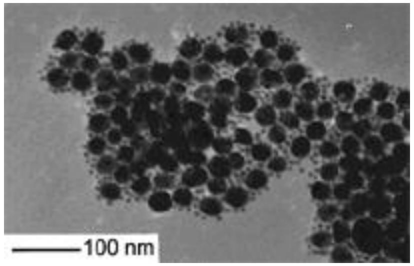
"Color change" of Au Nanoparticle







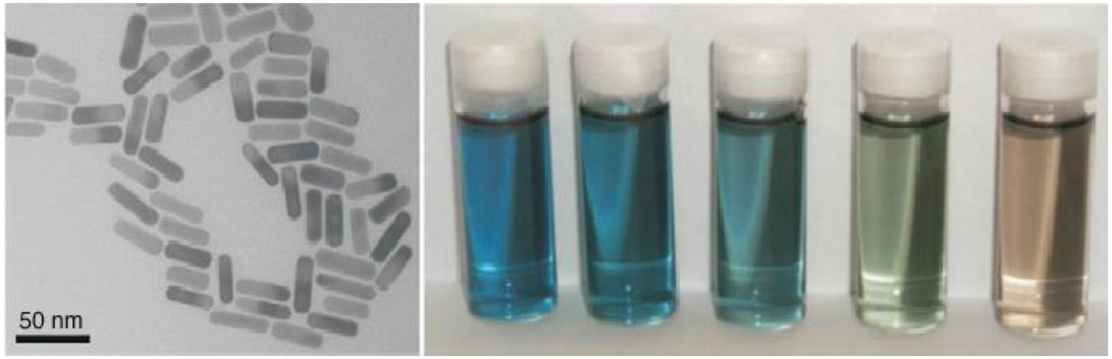
8 and 30 nm gold particles, no DNA link



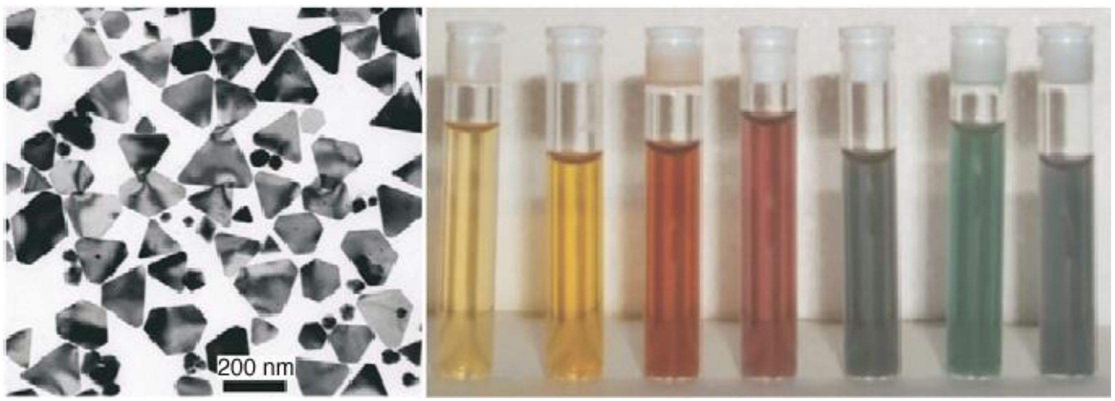
8 and 30 nm gold particles, DNA link

Chad A. Mirkin, *Inorg. Chem.* **2000**, *39*, 2258-2272

"Color change" of Au Nanoparticle



Au nanorods of increasing aspect ratio



Ag nanoprisms with increasing lateral size

Optical properties of metal nanoparticles depend strongly on particle size and shape and the distance to neighboring particles. For Au, Ag, and Cu the collective oscillation of surface conduction electrons in response to the alternating electric field of incident light induces a quantized charge density wave (plasmon resonance) that absorbs some visible wavelengths.

Semiconductor Nanocrystals "Quantum Dots"

Optical Properties

- Broad excitation profile
- Narrow emission bandwidth
- Good photostability
- Higher emission intensity