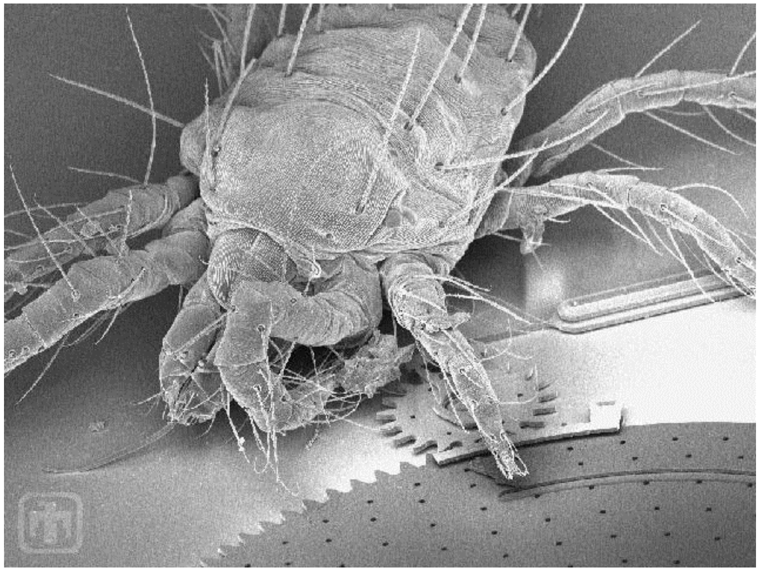
Scaling Law I

Why scaling is important at the micro and nano scale

- Micro and nano devices are orders of magnitude smaller than macro counterparts
- As the sizes shrink, different physical forces become more important or less important depending on their nature
- These changes dictate how the devoice must be built and what forces it will use to operate. This issue is called <u>scaling</u>.



Size and Shape

- In nature, the differences in sizes (for animals and plants) are also accompanied by the difference in shapes

Dik-Dik (5 Kg)

Dog

Buffalo (500 Kg)

Rhino

Elephant

- **Allometric Formula (Power Laws)**

$$y = bx^a$$

(power-law formulas)

- simply compare the relation of two measurements, ignoring all the complexities and subtleties of detailed changes in form.
- x and y are relative sizes of two parts in an organism
- a and b are constants

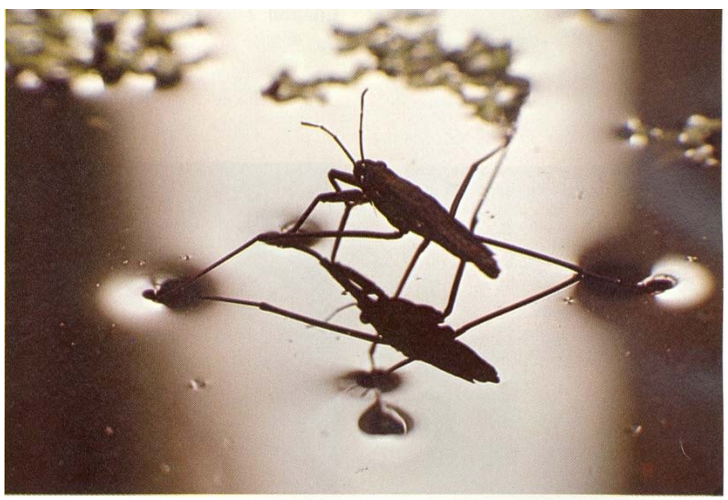
- What is the scaling law for the leg size (d) relative to the body size (L) (i.e. aspect ratio d/L)?

Isometry (geometric similarity)

Size change in geometrically similar living bodies has numerous effects and profound biological significance. Some of them are dependent on the length dimension (l), but most of them are dependent on the length dimension square (l^2), and a few are dependent on the length dimension cube (l^3)

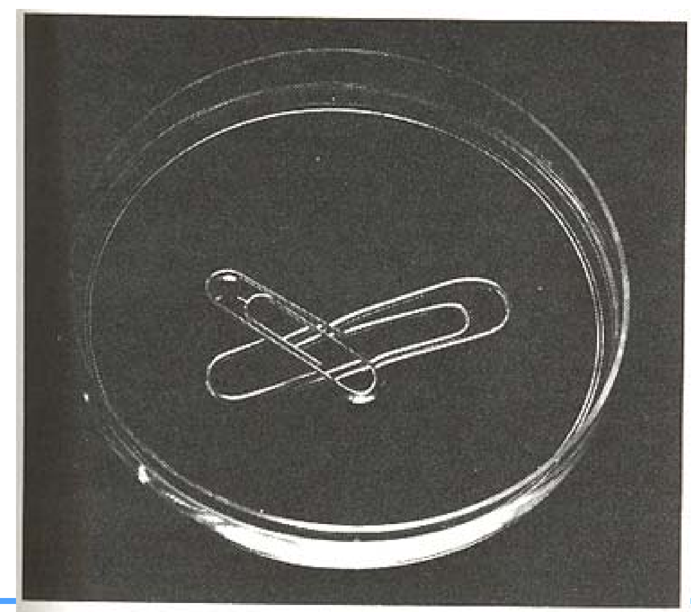
- Length (l)

the most obvious things that vary linearly are height or length (e.g. Height in trees, leg length in running animals)



A water strider depend on the forces of surface tension to support its weight.
Small insects of a suitable design can walk on water; while a large insect of the exactly the same design could not.

Surface tension is powerful to keep a small wax-coated paper clip floating at the surface of a dish of water, but a larger paper clip sinks.

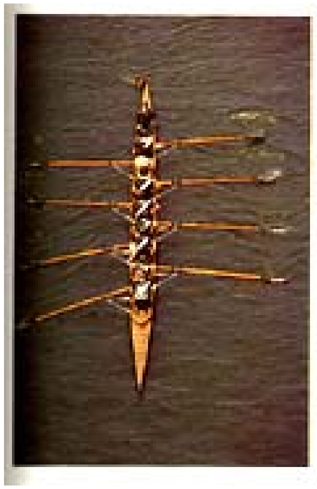


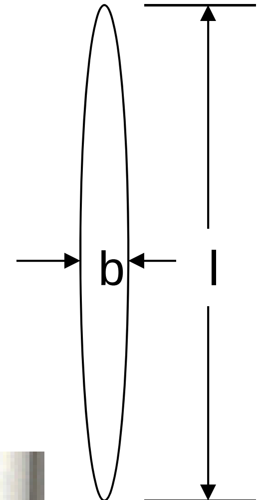
- Length Square (l^2)
  - The exchange of gases (e.g. oxygen and carbon dioxide) occurs through surfaces of the lungs, or gills or leaves (in plants)
  - In animal, the food is assimilated through the surface of the gut
  - Heat loss takes place through the skin
  - The strength of a bone or a muscle, or a insert structure of a tree is a function of its cross-sectional area
  - In locomotion, the drag of a swimming animal is, under certain conditions, proportional to its wetted surface area
- Length Cube (l^3)
  - The most significant factor is mass
  - Energy generation of an organism
  - The larger the animal or plant, the greater the significance of gravity
  - The buoyancy scales as l^3 means that a scaled-up floating body floats just as well as a smaller one.


Most mammals float with about the same fraction of their bulk out of the water. At the right is a floating sea otter, left are hippos

Why are there no mice is the arctic ?

Example 1: Racing Shells (the dependence of velocity on the length of the racing shell ?)





Shell dimensions and performances.

No. of oarsmen	Modifying description	Length, l (m)	Beam, b (m)	l/b	Boat mass per oarsman (kg)	Time for 2000 m (min) I	II	III	IV
8	Heavyweight	18.28	0.610	30.0	14.7	5.87	5.92	5.82	5.73
8	Lightweight	18.28	0.598	30.6	14.7				
4	With coxswain	12.80	0.574	22.3	18.1				
4	Without coxswain	11.75	0.574	20.5	18.1	6.33	6.42	6.48	6.13
2	Double scull	9.76	0.381	25.6	13.6				
2	Pair-oared shell	9.76	0.356	27.4	13.6	6.87	6.92	6.95	6.77
1	Single scull	7.93	0.293	27.1	16.3	7.16	7.25	7.28	7.17

- l/b ~ constant
- boat mass per oarsman~ constant
- Water drag : >90 % by skin drag at this speed (another energy loss is wave generation)

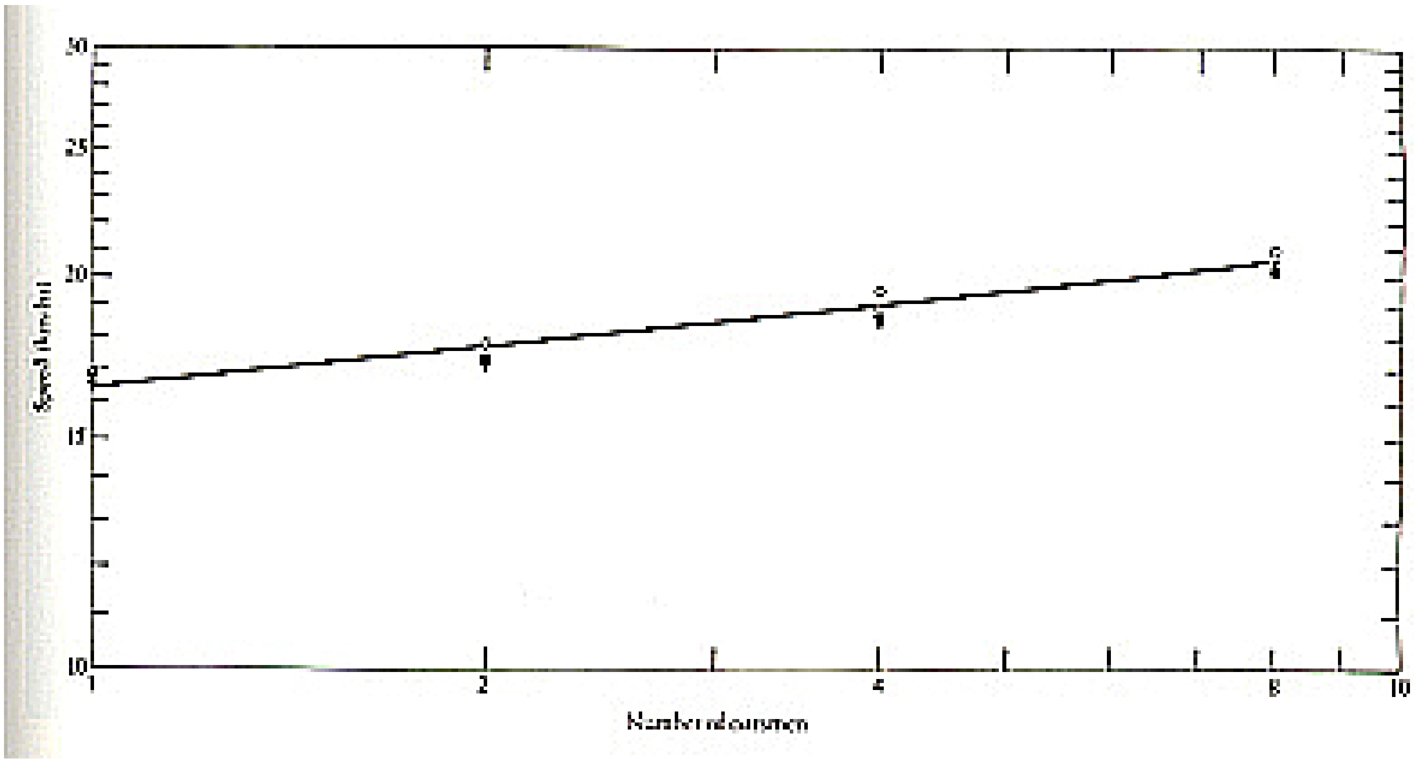
P_{req} = Drag $\times$ Velocity

$= k_1 (l^2 V^2) V$

$P_{avail} = k_2 \times$ (# of oarsmen ,n)

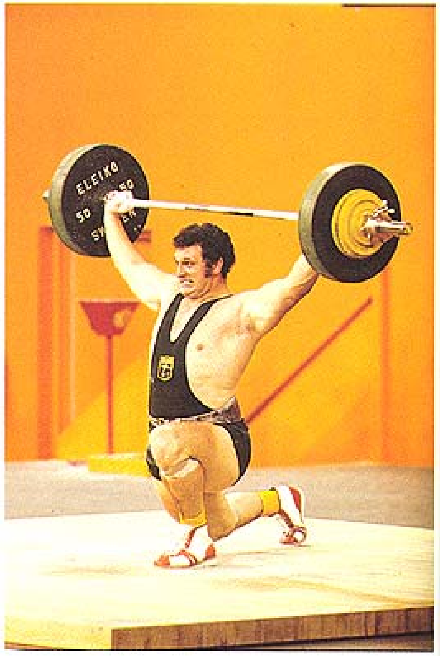
$\propto l^3$

$\Rightarrow V \propto l^{1/3} \propto n^{1/9}$



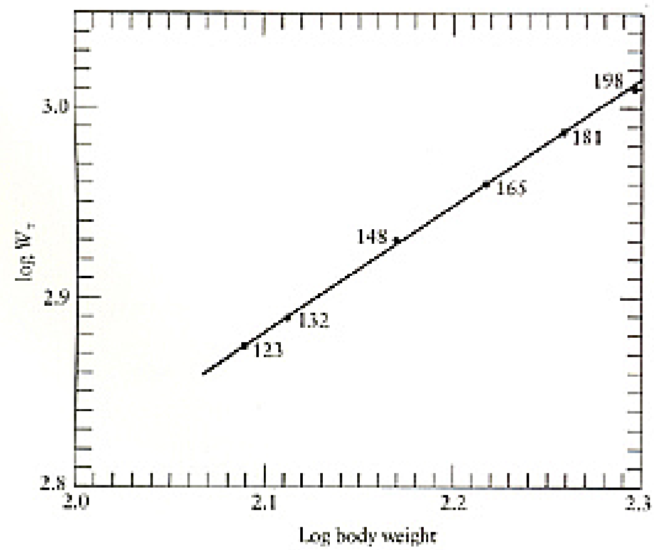
Average speed a 2000-m course for rolling shells seating one, two, four, and eight oarsmen.
Data from 1964, 1968 Olympics, and 1970 World Rowing Championships.

Example 2: Weight Lifting (What's the dependence of the maximum weight a human can lift on his/her body weight?)



According to the rule of isometry, for geometrically similar animals of the same mass density

- The body mass $m \propto l^3$;
- The surface area $\propto l^2$;
- Weight $\propto$ surface area $\propto (\text{mass})^{2/3}$



World weight-lifting record. The weight lifted (w_T) is precisely proportional to the 0.67 power of body weight

Microbial Movement

- Small -> •Avoid most problems related to gravity and inertial
  (New problems:)
  •Molecular cohesion (including surface tension)
  •Low Reynolds numbers

A world of low Reynolds number (e.g. $Re_{(bacteria)} \sim 10^{-6}$)

$$Re \sim \frac{\text{Inertial force}}{\text{Viscous force}}$$

- **Stopping Distance**
  The distance of gliding after propulsion stops
  - Pelican vs. Fly fishing
  - The stopping distance (gliding distance) is
    ~ 1/18 (Re • D)
    D: diameter of an object

e.g. a 2 μm bacteria at Re ~10^{-6} , the stopping distance is 10^{-11} μm.

- Clearly, bacteria don't move with the thrust-and-glide motions, characteristics of fish, some of which glide more than five body lengths between propulsive strokes.

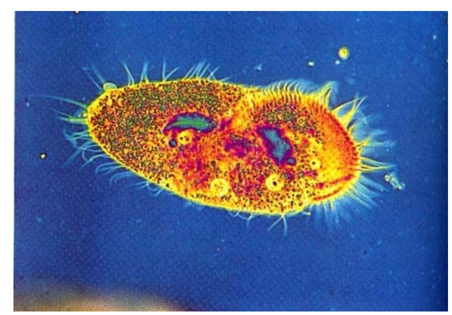
- # Reversibility

Video http://www.youtube.com/watch?v=p08_KlTKP50

Another import thing about swimming at low Reynolds number is reversibility.

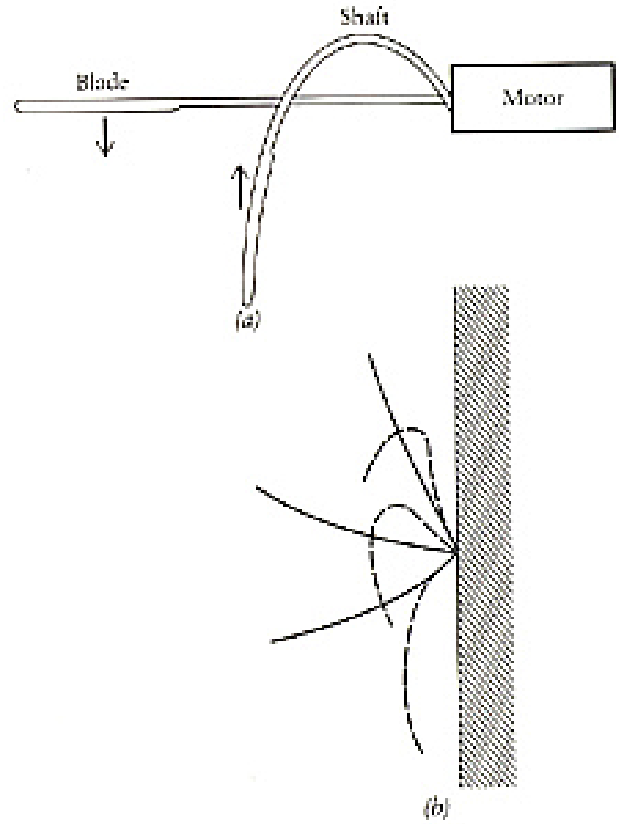
Reversibility of a low Reynolds-number flow. The narrow gap between the outer and the inner cylinders are filled glycerin. A blob of dyed glycerin is introduced. When the inner cylinder is turned in a direction the dye is mixed. When the cylinder is turned in the opposite direction the dye goes back to its original position.

- If a ciliated protozoan (~ μm) makes a motion with its cilia and then reverses those motion exactly, what happens ?
- How does a small organism swim ?



- Ciliary Propulsion

  - Mechanism : like a flexible oar
  - Organisms using ciliary propulsion with size 20 μm –2 mm
  - Regardless of organism size (including lung), typical length of cilia : 5-12 μm at 10 –20 Hz
  - Speed : ~ 1 mm/s

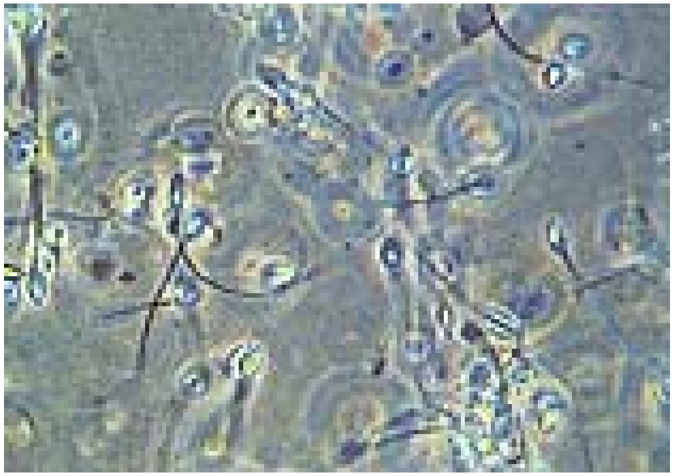


"Swimming" at the the low Reynolds number. A flexible oar remain rigid on the thrust stroke but bends significantly on the return stroke.

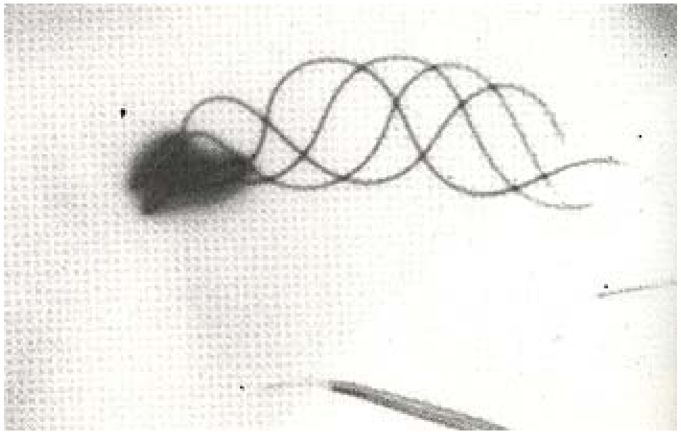
Flagellar Propulsion

http://www.youtube.com/watch?v=4hexn-DtSt4

- Mechanism : flagella function by means of the propagated waves generated
  - 2-D wave : like a wavy line (generated by beating the flagellum)
  - 3-D wave : like a corkscrew (generated by rotating the flagella, acting like a rotating propeller)
- Flagella : typically 0.2-1μm in diameter, 10-500 μm in length
- Speed : 0.1 to 0.2 mm/s, regardless of their size, which is only about 1/10 the swimming speed of ciliates.



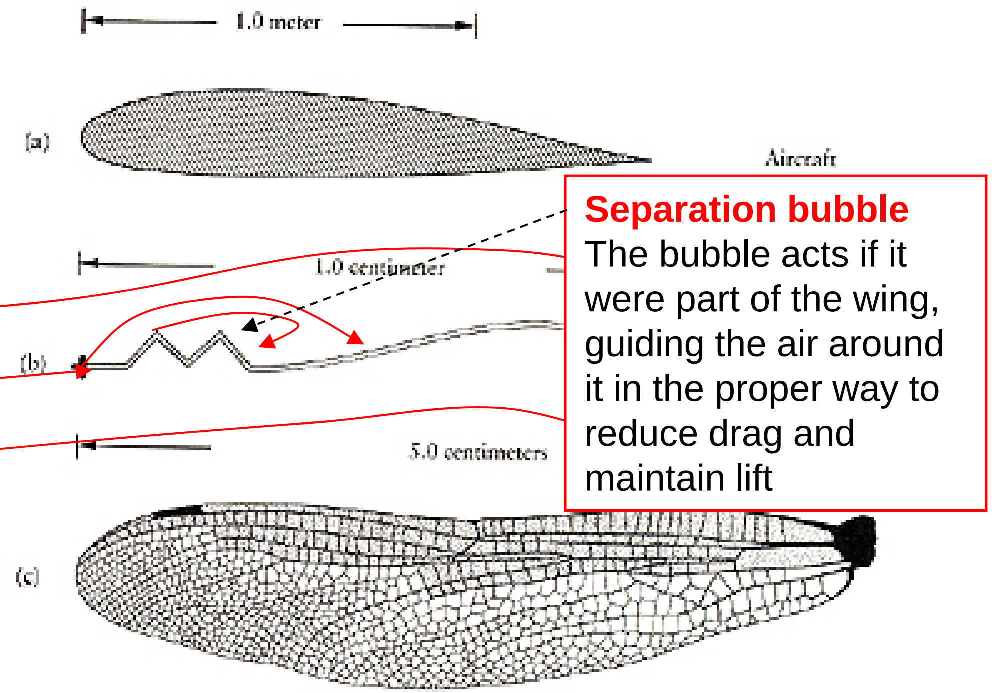
Human sperm cells. Each consists of a head attached to a whiplike tail, the flagellum. They swim by beating their tails from side to side.



Motions of sperm cell from a sea urchin

Insect Flight

- The flight muscles of insects are among the most active animal tissues. (transport of oxygen is through the air-filled tubes, or tracheae, instead of blood)
- As small as 1 μg with a wingspan of ~ 200 μm.
- The Reynolds numbers of hovering insects : ~ 1 to 10^4
- For airplane wing, it is ineffective when Re is below 10^5

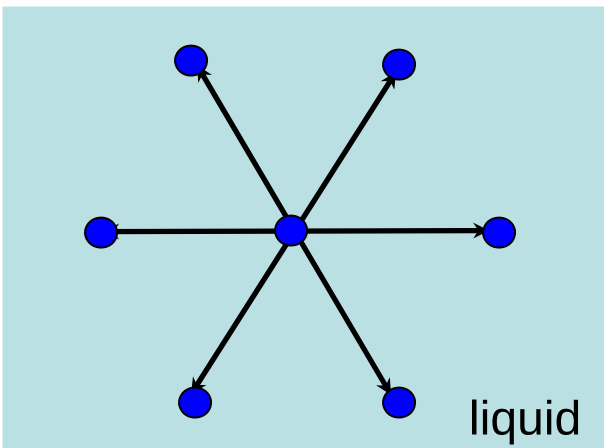


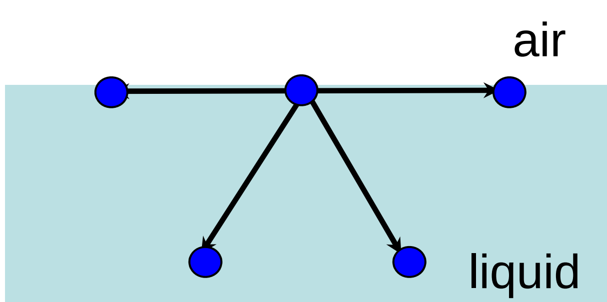
- Dragonfly wing :
  - (two horrible-looking) pleats
    - Increase Strength (like the corrugation design in a piece of cardboard);
    - poor aerodynamic properties ?
  - Lift-to-drag ratio
    The ratio of aircraft wing at the low Re : ~2 (it's ~ 30 at high Re)
    The ratio of dragonfly wing at the low Re: 12

A World Governed by Molecular Adhesion

- Large organisms – ruled by inertial and gravity
- Miniature organism – force and attraction between molecules (a variety of phenomena including surface tension) rule

Surface Tension





Surface tension occurs

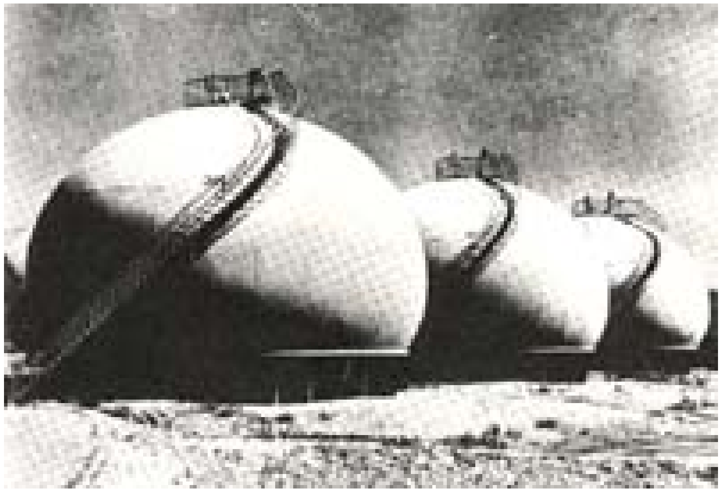
- The molecules on the surface tend to be pulled away from the surface, and therefore work must have been done to bring the molecules from the body of the liquid to the surface.
- The work required per unit area to bring molecules to the surface (I.e., to create a new surface) is called surface tension.
- Surface : work/area → it's dimension :force/length

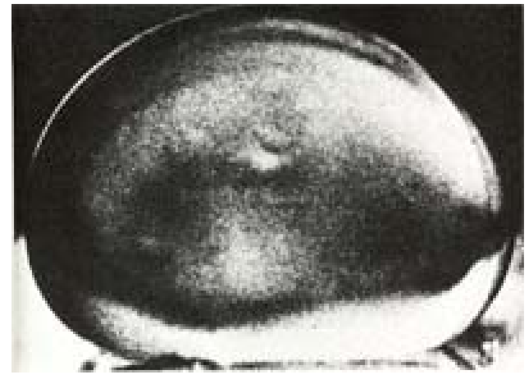
Young & Lapalce equation

$$\Delta P = \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

ΔP: pressure difference
γ:surface tension
R: radius of a surface

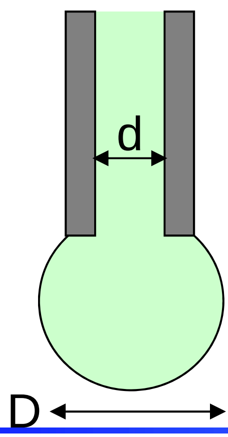
- Surface Tension and Gravity
  - Design of a liquid-storage tank





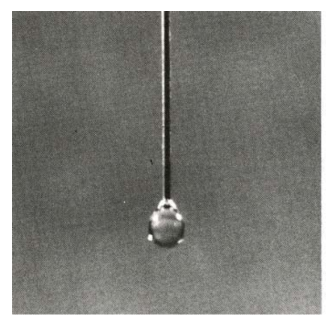
- Not required to endure bending stresses, only tensile stresses
- Uniform tension per length → no one region is likely to rupture than another

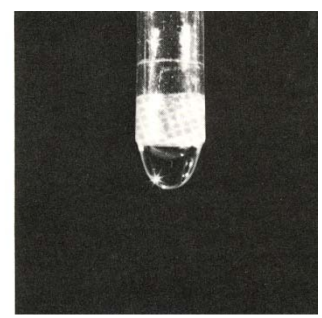
  - Size of a droplet

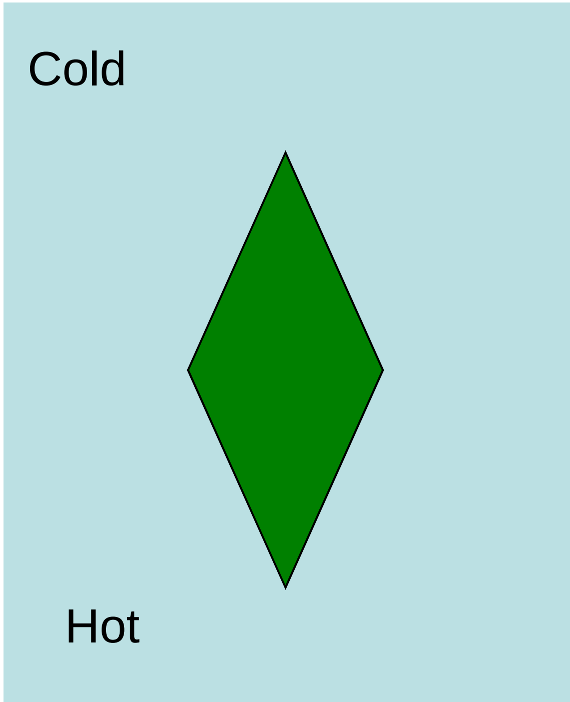


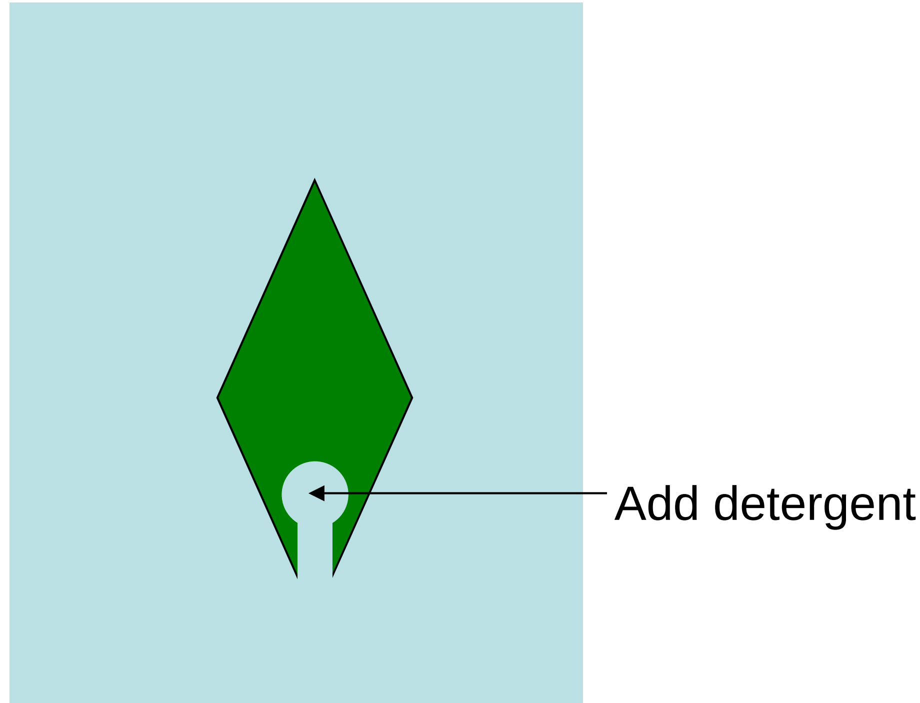
- $F_{surface\ tension} \propto d$
- $W \propto D^3$

⇨ $D/d \propto (1/d)^{2/3}$







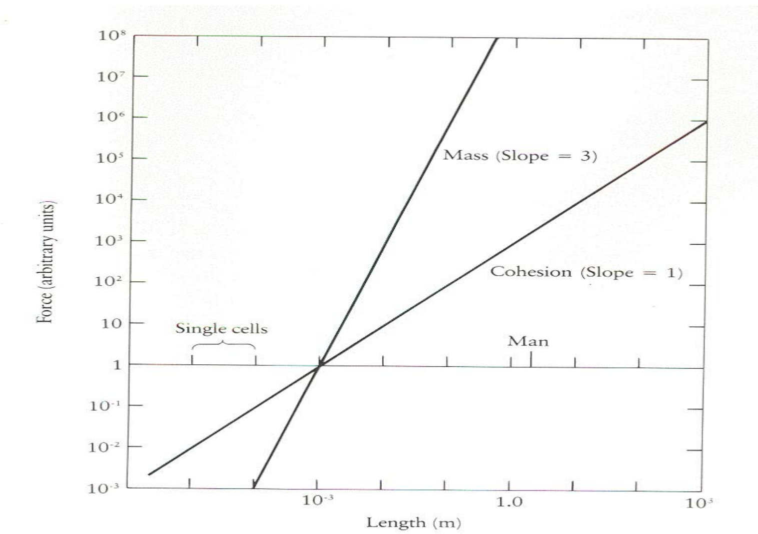


Which way the boat moves ?

- **Surface tension can be adjusted by**

1. Surfactants : surface tension is reduced with the addition of surfactant
2. Temperature: surface tension decreases as temp increases
3. Electric potential : a potential can alter (mainly decrease) surface tension

- Comparison of Inertial force and Cohesion



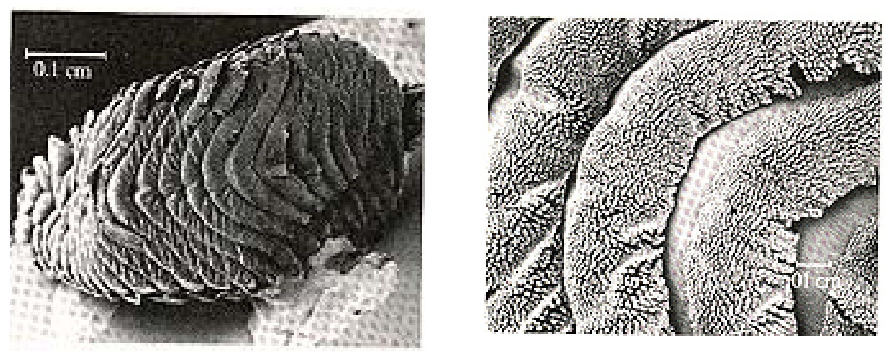
- Inertial $\propto$ L^3

- Cohesion $\propto$ L^1
(surface tension and molecular adhesion)

- The importance of gravity and cohesion are roughly equal at the size of ~ 1mm (e.g. fly can walk on wall)

- Beetle or Gecko on the Wall

A beetle has hairlike protuberances that allows close contact with rough surfaces



The toe pad of a gecko is constructed of a series of waves. Each wave is made of of tufts, like the tufts of a carpet