Stretching and Quenching of Flamelets in Premixed Turbulent Combustion

C. MENEVEAU* and T. POINSOT†
Center for Turbulence Research, Stanford University, CA

The stretch rate of flamelets in premixed turbulent combustion is computed using (1) detailed numerical simulations of vortex-flame interactions and (2) a model for intermittent turbulence taking into account all possible turbulence scales acting on the flame front. Simulations of interactions between isolated vortices and a laminar flame front are used to obtain a relation between the characteristics of a given vortex and the actual flame stretch generated by this structure. Quenching conditions and quenching times are also given by these simulations. A net rate of stretch is then defined in the case of a complete turbulent flow field as the difference between the total rate of flame stretch and the quenching rate due to scales that have a high enough energy and a long enough lifetime to quench locally the flame front. The net rate of stretch is computed for a variety of parameters of interest in practical applications. It is a function of the large-scale turbulence parameters and the laminar flame speed and flame thickness and may be used as an input in most flamelet models for premixed turbulent combustion. Different criteria for total flame quenching in premixed turbulent combustion are derived and compared (1) to the classical Klimov-Williams theory, (2) to a criterion proposed by Poinsot et al. [8, 9], who studied quenching according to the presence near the flame front of a single eddy able to locally quench combustion, and (3) to the experimental results of Abdel-Gayed and Bradley [6, 7].

1. INTRODUCTION

An important question in studies of premixed turbulent combustion is the determination of the combustion regime and the structure of the reaction flow. A common assumption is the ‘flamelet’ assumption [1, 2].

Under the flamelet assumption, chemistry is fast enough so that one can consider that the flow consists of two phases: the fresh gases and the burned gases. These two phases are separated by elements of flame called flamelets. In most flamelet models, one assumes also that each flamelet behaves like a laminar flame. This is not a necessary assumption: the only important assumption in flamelet modelling is related to the topology of the flow and to the fact that fresh and burned gases are separated by a relatively thin continuous region where chemical reactions take place. This region may have a laminar flame structure but it may also be thickened by small scale turbulence without invalidating the flamelet assumption.

The flamelet assumption is not always satisfied. Diagrams defining combustion regimes in terms of length and velocity scales ratios have been proposed by Bray [3], Barrere [4], Borghi [5], Peters [1], Williams [2], Abdel-Gayed and Bradley [6, 7] and Poinsot et al [8]. When the turbulence integral scale and the turbulent kinetic energy are known, these diagrams indicate whether the flow will contain flamelets, pockets, or distributed reaction zones. This information is essential for building a model for turbulent combustion. A continuous flame front, without holes, will not be modelled in the same way as a flame that is broken into many small pockets and where combustion does not take place along a sheet but in a more distributed manner. Under the flamelet assumption, a central parameter for turbulent combustion modelling is flame stretch. Flame stretch is a measure of the variations of the flame surface $A$ and is defined by [10, 11]

$$K = \frac{1}{A} \frac{dA}{dt}. \quad (1)$$

It is a local instantaneous characteristic of the...
flame front. Flame stretch controls the growth of the flame surface through two processes: (1) flame surface production and (2) flame quenching. Small to moderate flame stretch creates active flame surface while high stretch might lead to flame quenching.

1.1. Flame Surface Production

When the flamelet assumption is valid, the modelling of turbulent combustion mainly reduces to the evaluation of the flame surface density $\Sigma$ (defined as the flame surface per unit volume) [12–15] or the passage frequency of the flamelets [16–18]. For example, the formulation of the Coherent Flame model [12, 14] provides a conservation equation for $\Sigma$ in a Lagrangian frame moving with the turbulent flame:

$$\frac{d\Sigma}{dt} = \overline{K} \Sigma - Q_c,$$

where $\overline{K}$ is the mean stretch rate averaged along the flame surface. The second term $Q_c$ on the RHS of Eq. 2 corresponds to flame surface annihilation by mutual interaction of flame fronts (for example, the merging of two flame fronts [12]). The average stretch $\overline{K}$ is of utmost importance in Eq. 2 because it imposes the source term for the flame surface and therefore the mean turbulent reaction rate $\overline{w}$ given by

$$\overline{w} = w_L \Sigma,$$

where $w_L$ is the mean consumption rate per unit surface along the flame front (if one assumes that the flamelet has a laminar structure, $w_L$ will be the laminar consumption rate for the same chemical parameters and the same stretch [12]).

1.2. Flame Quenching and Definition of the Flamelet Regime

The most important mechanism controlling the validity of the flamelet assumption is the occurrence of flame quenching by turbulence. When no quenching occurs in a premixed turbulent flame, the flame zone is “active” everywhere and may be treated as an interface separating fresh unburned reactants from hot burned products. This regime is called “flamelet” regime.

It is necessary to discuss here the definition of a flamelet regime (Fig. 1). Poinsot et. al. [8] propose:

**Definition 1**

A premixed turbulent reacting flow is in a flamelet regime if, at any given time, any line connecting one point in the fresh gases to another point in the burned products crosses (at least) one active flame front, i.e. there are no holes in the active flame surface.

This definition allows for the existence of pockets of fresh gases in burned products as long as each pocket is surrounded by an active flame front. This mode of combustion is of the “corrugated flamelet” type [1].

Definition 1 is very restrictive. First, it is reasonable to assume that a hole that persists only for a short time will not force the flow to a “non-flamelet” regime. Second, even if the flame surface contains locally quenched surfaces, as long as these holes spread more slowly than the active flame surface, the regime will correspond to partial quenching and the flamelet approach will still provide a reasonable estimate of the reaction rate if quenching is accounted for. As we are interested in developing models for engineering applications, it is convenient to relax Defini-
tion 1 and to introduce a broader definition of the flamelet regime:

**Definition 2**

A premixed turbulent reacting flow is in a flamelet regime if holes (generated by local quenching of the flame front) do not inhibit the growth of the active flame surface.

Definition 2 allows us to consider regimes of partially quenched flames as flamelet regimes. What happens when holes in the flame front grow fast enough to interfere with the active flame surface is an open question. Fresh and hot gases will diffuse before they react and our definition of flamelets will break down. In this case, it is possible that the flow will still be able to sustain combustion in a regime called distributed reaction zones. However, it might also be driven to total quenching. This point cannot be asserted at the present time. Throughout this article, we call this limit global (or total) quenching although it might, in fact, be only a transition to another regime of combustion (without flamelets).

Local flame quenching occurs when the flame front is submitted to external perturbations like heat losses or aerodynamic stretch which are sufficiently strong to decrease the reaction rate to a negligible value or in some cases to completely suppress the combustion process.

Quenching in laminar flames has received considerable attention in the last years. Asymptotic studies of laminar stagnation point flames established by the counterflow of reactants and products [19-21] reveal that a laminar flame can be quenched by stretch if the flow is nonadiabatic or if the Lewis number (defined as the ratio of the thermal diffusivity to the reactant diffusivity: \( Le = \lambda / (\rho C_p D) \)) is greater than unity. These results have been confirmed by numerical methods for simple or complex chemistry [22, 23] and by experimental studies [24, 25, 26].

The idea that quenching mechanisms evidenced in laminar flames may be responsible for partial or total quenching in premixed turbulent flames is an important ingredient of many models of turbulent combustion [1, 12]. Experiments show that quenching can, indeed, occur in turbulent combustion [6, 7]. However, the prediction of quenching in turbulent flames is still an open question. The classical theoretical approach to predict quenching in turbulent flames is to assume that flamelets behave like laminar stagnation point flames [3] and are quenched for similar critical values of stretch. This is a crude approximation. In laminar stagnation point flames, a constant steady stretch is imposed to a planar flame. In a turbulent reacting flow, flamelets are stretched by vortices. Therefore, the stretch they experience is changing with time because the vortices are convected by the mean flow and are dissipated by viscous effects. Flamelets are also free to move to escape from regions of high stretch (which is not the case for laminar stagnation point flames). Moreover, vortices curve the flame front, making the analogy between flamelets and planar stagnation point flames doubtful. These points suggest that information on quenching in laminar stagnation point flames are not relevant to predict quenching in turbulent flames. A central difficulty to improve on this classical approach is the estimation of the flame stretch \( K \) in a turbulent flow.

### 1.3. The intermittent Turbulence Net Flame Stretch (ITNFS) Model

From the previous discussion, it is clear that the mean value of the flame stretch \( \bar{K} \) is an essential parameter in turbulent combustion. It controls flame quenching as well as flame surface creation. It is also clear that studies of laminar stagnation point flames cannot be used directly to study quenching or flame surface creation in a turbulent flow. Additional parameters like curvature, viscous dissipation, and thermodiffusive effects also have to be considered.

Different expressions may be found in the literature for the mean flame stretch \( \bar{K} \). Bray [3] and Cant and Bray [17] propose

\[
\bar{K} = \sqrt{\epsilon / \nu} ,
\]

where \( \epsilon \) is the dissipation of turbulent kinetic energy and \( \nu \) is the kinematic viscosity

Candel et al. [12] use

\[
\bar{K} = \epsilon / k ,
\]

where \( k \) is the turbulent kinetic energy. We give here a more precise estimate of the flame stretch by combining different approaches:

1. Use direct simulations of flame–vortex interactions to predict the effect of a given isolated
structure on a laminar flame front. Using results on flame–vortex interaction allows us to take viscosity, curvature, and transient effects into account. The basis for these results is the work of Poinsot et al. [8], which we describe in Section 2.

2. Use detailed experimental data about intermittent turbulence to determine the distribution of stretch along the flame front [46]. This approach is described in Section 3.

3. Define a net stretch of the flame by subtracting the rate of destruction of existing flame surface by quenching from the rate of increase of surface due to hydrodynamic straining.

The idea behind the ITNFS model is to use a complex model to describe the interaction of one vortex with a flame front and to extend it to a complete turbulent flow by supposing that the total effect of all the turbulent fluctuations can be deduced from the behavior of each individual scale in the fresh gases. This is clearly an important approximation. First, the interaction of multiple scales with the flame front cannot, in the general case, be reduced to the sum of the interactions of each vortex with the flame. Nonlinear effects are to be expected. Second, the flame is not affected only by the fluctuations present in the stream of fresh gases. Flame-generated turbulence can also play a role. Reignition of fresh gases crossing a locally quenched flame front may also be an important mechanism. Therefore, the present approach should be viewed only as a first step towards a more complete treatment of the turbulent reacting flow. However, it represents a substantial improvement on classical estimates of the flame stretch [3, 12, 17]. The ITNFS model may be used in any flamelet model [12, 16, 17].

2. DIRECT SIMULATION OF TWO-DIMENSIONAL FLAME–VORTEX INTERACTION

2.1. Principle

A simple approach to understand the combined effects of stretch, curvature, viscosity and transients in turbulent combustion is to perform computation of the interaction between deterministic vortices and a laminar flame front [8, 27–29, 32–34, 38]. For this study, we use results obtained by Poinsot et al. [8].

In this work, the interaction of a vortex pair (characterized by a speed $v_r$ and a size $r$) with a premixed laminar flame front (characterized by its laminar unstretched flame speed $s_L$ and its laminar flame thickness $l_f$) is considered (Fig. 2).

The original characteristic of this work is to include the fluid-mechanical and chemical mechanisms required to produce flame quenching as suggested by asymptotic studies [20, 21]. A simple one-step chemistry with an Arrhenius law is used to describe combustion. The formulation is fully compressible, allowing for variable density, realistic heat release, and dilatation through the flame front. The Lewis number is 1.2, the viscosity and diffusion coefficients are changing with temperature and a simple linear model for radiative heat losses is included (similar to models used in asymptotic studies [2]. Poinsot et al show...
that the value of the Lewis number has a small effect on quenching properties when the heat losses are strong [8]. Heat losses are fixed to a high value to promote quenching: the temperature decreases by 15% at a distance of 3 flame thicknesses downstream of the reaction zone. This leads to a maximization of quenching effects and we reckon that some of the results of this paper might change if the heat losses or the Lewis number are changed. We do not feel that this issue is important because the outcome of this analysis (the net flame stretch) is to be used with average-based turbulent combustion models whose accuracy level is probably lower. The computation is performed on a two-dimensional grid encompassing 25,000 points using a high-order finite difference scheme [36]. A complete description of the results may be found in [8]. We will use them to obtain three types of information:

1. What is the actual stretch sensed by the flame front when it is submitted to a given vortex?
2. What are the conditions required for a given vortex to quench the flame front? (These instantaneous conditions will be called "quenching conditions." )
3. When quenching conditions are satisfied, how much time must they hold before actual quenching is observed?

Point 1 (the measurement of the characteristic time of the stretching process) is addressed in Section 2.2. Points 2 and 3 (the quenching conditions and characteristic time) are considered in Section 2.3.

2.2. Flame Stretching by a Vortex: The Efficiency Function

Direct simulation was used in [8] to construct a spectral diagram (Fig. 3) that describes the result of the interaction between a laminar flame and a vortex pair in terms of the scale ratio $r/l_F$ and velocity $v_r/s_L$. In this diagram, a cutoff limit was defined. Vortex pairs located below the cutoff limit correspond to scales which do not modify the total reaction rate (or the total flame surface) by more than 5%. Kolmogorov scales are below the cutoff limit, indicating that these scales have no effects on the flame front, mainly because their lifetime is too short. This is an important result because a usual assumption is that Kolmogorov scales generate the highest strain rates. However, despite their high theoretical strain, these structures have very little effect on the flame front. This result implies that the relation between the theoretical strain of a structure and the actual stretch which this structure induces on a flame is more complex than usually thought.

For the present study, this relation was explicitly derived by introducing an efficiency correction function $C$: consider a vortex pair with known characteristics (namely its size $r$ and its speed $v_r$) and define the actual flame stretch.
\[
\frac{1}{A} \frac{dA}{dt} = C \left( \frac{r}{l_F} \right) \frac{\nu_r}{r},
\]

where \( C \) accounts for the fact that the strain \( \nu_r / r \) generated by the vortex is not entirely converted into effective flame stretch. This is due, either to viscous effects (the vortex is dissipated by viscosity before it can affect the flame), to curvature effects (the small size of the vortex induces local thermodiffusive effects which inhibit the stretching of the flame surface) or to geometrical effects (a very small vortex will not be able to stretch a large flame front). The efficiency function is probably situation dependent. Because the work of Poinsot et al. was performed in the case of vortices impinging at normal incidence on a flame front, the present efficiency function is valid for flames propagating into a turbulent flow with zero mean speed (like in a piston engine) but further studies may be needed to determine \( C \) in other situations (for example, in the case of a turbulent flame stabilized behind a bluffbody.) To first order, however, the present estimate is probably well suited to most situations.

The function \( C \) was estimated from direct simulation results in the following way. Consider a typical flame–vortex interaction with \( r/l_F = 5 \) and \( \nu_r/s_l = 28 \). Instantaneous fields of fuel mass fraction at three different instants are displayed in Fig. 4. The time variations of the total reaction rate \( Q \) in the control volume are also shown (The size of the control volume is chosen to be proportional to the size of the vortex pair.) Three successive steps can be observed. First (phase I), depending on the initial position of the vortex, an induction phase takes place during which the vortex pair moves and enters the influence zone.
FLAMELET STRETCHING AND QUENCHING

of the flame front. Second (phase II), the vortex pair starts stretching the flame. During this phase, the total reaction rate grows with time. If the vortex was not affected by viscous effects and if the flame was infinitely thin, this growth would be exponential. However, in most computations, it is only linear. No quenching occurs during this phase. Because the estimation of the flame surface is a costly and imprecise operation, we have assumed that the growth of the total reaction rate \( Q \) in the computation domain was directly related to the growth of the flame surface \( A \) and therefore, estimated the flame stretch \( (1/A)(dA/dt) \) during phase II by \( 1/(Q(t = 0)(dQ/dt)) \). Finally in the last phase (III), the flame fronts interact and merge, leading to flame surface consumption by mutual annihilation. (We do not use this information here. It would be needed to model the consumption term \( Q_c \) in Eq. 2).

The efficiency function \( C \) is plotted versus \( r/l_F \) in Fig. 5. It is strongly dependent on the ratio of vortex size and flame thickness \( r/l_F \), and is roughly independent on the velocity ratio \( v_r/s_L \). For low values of \( r(r/l_F < 2) \), the conversion of vortex strain into actual flame stretch is very small. It reaches (asymptotically) unity when the vortex is larger (typically when \( r/l_F > 5 \)). This asymptotic limit corresponds to situations where the flame front may be viewed as a material surface, with zero thickness \( (r/l_F \rightarrow \infty) \). Material surfaces for which the efficiency function is unity represent interesting cases because they are a limit case of our derivation and have been extensively studied by Pope and his coworkers [41, 42].

A simple curve fit was used to approximate \( C \):

\[
C \left( \frac{r}{l_F} \right) = 10^{-c(s)},
\]

where

\[
c(s) = \frac{0.545}{s + 0.364}
\]

and

\[
s = \log_{10} \left( \frac{r}{l_F} \right)
\]

This is shown as the solid line in Fig. 5.

2.3. Flame quenching

In Ref. 8, a quenching limit was also defined (Fig. 3). Vortex pairs located above the quenching limit are capable of quenching the laminar flame front and lead to the formation of a pocket of fresh gases located in the burnt gases but surrounded by a quenched flame front. The pocket of fresh gases does not reignite in the hot gases because of the presence of heat losses. This situation would not correspond to a flamelet regime as defined above.

The quenching limit gives us an estimate of the quenching conditions, i.e. of the length and velocity scales needed to quench a flame front. However, we also need to know how long such perturbation scales must be applied on the flame front before actual quenching takes place. A quenching time \( t_q \) may be defined as follows (Fig. 6): the flame speed \( s_{axis} \) on the symmetry axis is computed at each time using the integral of the reaction rate along the axis. (This case corresponds to the one presented in the previous section (Fig. 4) and instantaneous fields of reaction

---

**Fig. 5.** The efficiency function \( C(r/l_F) \) to estimate the effective stretch.
Fig. 6. Example of measurement of the quenching time ($r/l_F = 5$ and $v_r/s_L = 28$).

Fig. 7. The correlation between quenching time, vortex time (vortex strain $Vr/r$) and flame time.
rate are displayed in Fig. 6). The flame speed $s_{\text{axis}}$ decreases with time and reaches zero after a time equal to the quenching time $t_q$ (in practice, the tails of the flame may reach the symmetry axis before the flame tip is completely quenched, see Fig. 6; in this case $t_q$ is estimated using the slope of the curve $s_{\text{axis}}$ versus time.) Note also that the limit between Phases I and II are somewhat arbitrary: the flame speed $s_{\text{axis}}$ starts decreasing before the end of Phase I. This shows that, in the first instants of the interaction, the local flame speed is affected before a significant modification of the total reaction rate can be noticed. The quenching time $t_q$ is then plotted versus the initial vortex characteristic strain $v_r/r$ for different vortex–flame computations leading to quenching. Results (Fig. 7) show that $t_q$ is quite different from the vortex characteristic time $r/\nu_r$ and remains close to the flame time $l_F/s_L$.

In other words, however strong the vortex strain may be, the time required to reach quenching is the same and is fixed by the flame characteristics $l_F$ and $s_L$.

### 3. INTERMITTENCY IN NONREACTING FLOWS

The previous section describes the effect of one isolated vortex pair on a flame front. To address the problem of turbulent combustion, we have now to consider the effects of a complete turbulent flow field on a flame front. The principle of the ITNFS model is to take into account the existence of a wide range of scales as well as the statistical distribution of the velocity of each scale of motion, at a given time in the vicinity of the flame front. This is done by using a model for intermittent turbulence which is first described here for nonreacting flows.

The scales to be considered ($r$) are assumed to be smaller than the integral scale of turbulence ($L$) and larger than the Kolmogorov scales ($\eta_K$). It will be assumed that all scales follow scaling laws that strictly speaking are valid in the inertial range only. If the turbulence was non-intermittent, then there would be a single velocity $\nu_r$ at each scale $r$, given by [47]

$$\nu_r \approx u' \left( \frac{r}{L} \right)^{1/3}.$$  \hspace{1cm} (10)

Here $u'$ is the rms velocity of the turbulent flow and $L$ is the integral scale of turbulence. However, the intermittent nature of turbulence implies that there is a distribution of velocities that an eddy of size $r$ can take on. The local characteristic velocity $\nu_r(x)$ at some snapshot of time can be related to the local rate of dissipation of turbulent kinetic energy by

$$\nu_r(x) = \left[ r\epsilon_r(x) \right]^{1/3},$$  \hspace{1cm} (11)

where $\epsilon_r(x)$ is the rate of dissipation averaged over a region of size $r$ centered around position $x$. We remark that Eq. 11 is not an exact relation for turbulence. It is derived from dimensional analysis applied locally to a region of size $r$, and invoking the usual Kolmogorov picture of local interactions in the energy cascade. In Ref. 46 it was shown by comparison with the results of Ref. 51 that positive moments of $\nu_r(x)$ scale in the same fashion as $(\nu^2)$. This is a strong indication that Eq. 11 is correct, at least when $\nu_r$ or $\epsilon_r$ exhibit values above their respective mean. Since these are the more relevant values for the analysis to follow, we assume that Eq. 11 is essentially correct. The goal of this work is to incorporate detailed knowledge of the statistics of $\nu_r$ in the calculations relevant to turbulent combustion.

The statistics of $\nu_r$ can be related to the statistics of $\epsilon_r$ through Eq. 11, this being useful because much is known about the probability distribution function of $\nu_r$ as a function of $r$. Here, we briefly review the statistical and geometrical characterization of the rate of energy dissipation in high-Reynolds-number turbulence.

A well-known approach [17, 52] is to assume that the dissipation $\epsilon(x)$ is lognormally distributed [48, 49]. However, we need also the distribution of the locally averaged values $\epsilon_r(x)$ for any value $r$ within the range of interest ($\eta_K < r < L$). Even though this can also be addressed directly by the formalism of Refs. 48 and 49, we feel that it is clearer to use the so-called multifractal formalism, of which the lognormal distribution is only a special case. For a detailed account on the subject, see references [43–46]. In the multifractal description of turbulence, one focuses on a local exponent $\alpha(x)$ relating the locally averaged values $\epsilon_r(x)$ to the global mean value of dissipation rate $\langle \epsilon \rangle$

$$\epsilon_r(x) = \langle \epsilon \rangle \left( \frac{r}{L} \right)^{\alpha(x)-3}.$$  \hspace{1cm} (12)
The probability density of $\alpha$ typically obeys

$$P_r(\alpha) = \frac{\sqrt{f^*(\alpha) \ln \left(\frac{L}{r}\right)}}{2\pi} \left(\frac{r}{L}\right)^{3-f(\alpha)}.$$  

(13)

Here, $f(\alpha)$ is an $r$-independent function (it is also independent of the Reynolds number of the flow), and essentially corresponds to a distribution function of the dissipation in log-log units, properly normalized by $\log(r/L)$. Now we recall that for a fractal set embedded in three-dimensional space, the probability that a cube of size $r$ contains parts of the fractal goes like $P_r \sim r^{3-D}$, where $D$ is the fractal dimension. From Eq. 13, we see that $f(\alpha)$ can also be interpreted as the fractal dimension of the regions in space where $\alpha$ has a certain value (at a given instant in time). We point out that this geometrical interpretation of $f(\alpha)$ will not be crucial in the context of this work.

The function $f(\alpha)$ has been measured in detail in a variety of turbulent flows [43, 45, 46] and was shown to be a universal feature of high Reynolds number turbulence within the experimental accuracy. It is shown in Fig. 8. The usual log-normal approximation [48] can be shown to be an expansion of $f(\alpha)$ about its maximum up to second order. It corresponds to

$$f(\alpha) = 3 - \frac{1}{2\mu} \left(\alpha - 3 - \frac{\mu}{2}\right)^2,$$

(14)

where $\mu$ is the intermittency exponent. The value $\mu = 0.26$ gives the dashed line in Fig. 8, which for present purposes is a reasonably good fit to the distribution. It follows that the distribution function of $\alpha$ is given by

$$P_r(\alpha) = \sqrt{\frac{\ln \left(\frac{L}{r}\right)}{2\pi \mu}} \left(\frac{r}{L}\right)^{1/2\mu(\alpha-3-\mu/2)^2}.$$  

(15)

This now allows to compute the desired statistical properties of $\epsilon_r$ [as well as of $V_r(x)$] for any value of $r$ ($\eta_K < r < L$).

![Fig. 8. Shows $f(\alpha)$ vs $\alpha$ curve of the dissipation field in high Reynolds number turbulence. $\alpha$ is the local exponent defined by Eq. 12. $f(\alpha)$ is the probability density function of $\alpha$ defined by Eq. 13. The symbols are results from a variety of turbulent flows (laboratory boundary layer, grid turbulence, wake behind a cylinder and atmospheric surface layer). The dashed line is the log-normal approximation with the intermittency exponent $\mu = 0.26$.](image-url)
4. THE ITNFS MODEL

This section describes the calculation of the rate of net flame surface production (or destruction), referred to as the Intermittent Turbulence Net Flame Stretch model. First we consider the total rate of stretch induced by all the eddies that act on the flame. Then we consider the fraction of existing flame surface being destroyed by all the eddies that can produce quenching. These results are used to define a net stretching rate and to derive criteria for global quenching.

4.1. The Stretch of Flame Surface Without Quenching

A possible estimate for the stretch induced by turbulence is the one corresponding to the smallest scales, the Kolmogorov scale $\eta_K$. This can be written either as $\sqrt{\langle \epsilon \rangle} / \nu$ or $\nu_{nk} / \eta_K$, where $\nu$ is the kinematic viscosity of the fluid and $\nu_{nk}$ is the Kolmogorov velocity. Another estimate uses the large scales, and obtains $u'L$, or $(k) / k$, where $k$ is the kinetic energy. In general, the local flame stretch at a location $x$ of the flame front, induced by eddies of size $r$ will be

$$ S_r(x) = \frac{v_r(x)}{r}. $$

Using Eqs. 11 and 12, $S_r(x)$ can be written in terms of the local exponent $\alpha(x)$ as

$$ S_r(x) = \sqrt{\frac{\langle \epsilon \rangle}{\nu}} \Re_L^{-1/2} \left( \frac{r}{L} \right)^{-2/3 + (\alpha(x) - 3)/3}. $$

Here the Reynolds number $\Re_L$ is defined as $\Re_L = u'L/\nu$.

Equation 17 reduces to $S_{\eta K} = \sqrt{\langle \epsilon \rangle} / \nu$ if there was no intermittency (that is $\alpha = 3$ everywhere) and if one considers the strain to be dominated by the smallest scale $r = \eta_K$. (As indicated in Section 2.2, the fact that scales close to $\eta_K$ dominate the strain does not mean that they will also control flame stretch. Only in the asymptotic case of material surface for which the efficiency function $C$ is unity, will the small scales control flame stretch.)

The strain rate $S_r(x)$ will be converted into actual flame stretch with an efficiency defined by the function $C$, i.e., the flame stretch at a point $x$ on the flame front, created by vortices of size $r$ will be

$$ K_r(x) = c_{ms} C \left( \frac{r}{l_F} \right) S_r(x), $$

where $K_r(x)$ is given by Eq. 18 and $P_r(\alpha)$ by Eq. 15. Here we are making the tacit assumption that the statistics of $\alpha$ along the flame front is the same as everywhere else in the turbulent flow. That is, we assume that the combustion process does not influence the turbulence itself. Naturally though, the combustion is allowed to influence the detailed interactions between a given eddy with the flame, as embodied in the efficiency function $C(r/l_F)$.

The next question concerns the limits of integration. Eq. 12 shows that high values of $\alpha$ correspond to low values of the eddy velocity, and vice-versa. The limit $\alpha = -\infty$ corresponds to very strong eddies. The other limit ($\alpha = \infty$) corresponds to the weakest eddies, essentially with zero velocity. The experimental results of Meneveau and Sreenivasan [46], as well as other plausibility arguments suggest that $\alpha$ is bounded between some values $\alpha_{\min}$ and $\alpha_{\max}$. This can be seen from Fig. 8, where it is apparent that the experimental results fall off faster than the log-normal approximation. However, at the level of accuracy intended for this work, this distinction is
irrelevant because our results are not influenced by details of the tails of \( f(\alpha) \).

Carrying out the integration of Eq. 19 over all \( \alpha \) (i.e., along the flame front) yields the mean flame stretch \( K_r \) generated by scales of size \( r \):

\[
\langle K_r \rangle = c_{ms} C \left( \frac{r}{l_F} \right) \left[ \frac{\langle \varepsilon \rangle}{\nu} \right] \text{Re}_L^{1/2} \left( \frac{L}{r} \right)^{2/3-\mu/9}.
\]

For the efficiency function, we use the regression given by Eq. 7. It is important to stress that the integral over \( \rho \), as opposed to the integral over \( \alpha \), is not over a normalized probability distribution of scales. This is because at a given point, the action of all scales can be felt simultaneously, and not in an exclusive fashion of one scale at a time (if one were to consider one scale at a time, the present argument would give a probability density of scales proportional to \( 1/r \), as used for instance in Ref. 50).

The integration over \( p \) has to be performed for scales between the smallest scale at which the eddies can stretch the flame (which according to Fig. 5 is roughly \( 0.44 I_F \)) and \( L \). If, however, the Kolmogorov scale \( \eta_K \) is larger than \( 0.44 I_F \), then the integral should only be performed down to scales equal to \( \eta_K \). This means that we take \( p_{\min} = 0 \) and \( p_{\max} = \ln(L/0.44 I_F) \) or \( p_{\max} = \ln(L/\eta_K) = 3/4 \ln(\text{Re}) \), whichever is smaller. This allows to evaluate the integral in Eq. 22 for any desired pair of values \((u'/s_L, l_F/L)\). The integration is done numerically. Figure 9 shows the ratio

\[
\gamma_k = \frac{\langle K \rangle}{\sqrt{\langle \varepsilon \rangle/\nu}}
\]

as a function of \( L/I_F \) for a variety of values of \( u'/s_L \). It can be seen that \( \sqrt{\langle \varepsilon \rangle/\nu} \) overestimates the actual strain felt by the flame in all of the parameter space. Only when \( u'/s_L \) is very small (pseudo-laminar flames), or when \( L/I_F \) is very

\[
\text{Fig. 9. Total stretch } \langle K \rangle \text{ acting on the flame normalized with the strain of the smallest scales of the flow } \left( \langle \varepsilon \rangle/\nu \right)^{1/2}. \text{ } L/I_F \text{ is the ratio of the integral turbulent scale } L \text{ to the flame thickness } l_F \text{ and } u'/s_L \text{ is the ratio of the RMS turbulent velocity } u' \text{ to the laminar flame speed } s_L.
\]
large (material surfaces) does \( \gamma_K \) tend to the constant \( c_{ms} \) which is of order unity. In fact, the constant \( c_{ms} \) may be determined now using the results of Yeung et al. [42]. These authors have studied the stretch of material surfaces in isotropic turbulence. In this case, the efficiency function is unity and their results show that the stretch is directly related to the characteristic strain at the Kolmogorov scale through the relation \( \langle K \rangle = 0.28 \sqrt{\langle e \rangle}/\nu \) so that the limit value of \( \gamma_K \) when \( L/l_F \) is very large, should be 0.28. Therefore, we have chosen \( c_{ms} = 0.28 \). (A similar result for the strain of premixed flames was obtained by Cant and Rutland [39] in the case of a flame with finite thickness submitted to large scale turbulence.)

Physically, for a fixed \( L/l_F \), increasing \( u'/s_L \) amounts to increasing the Reynolds number of the flow. This is turn implies a decrease in the Kolmogorov scale which becomes smaller and smaller as compared to the flame thickness. Therefore, with increasing Reynolds number, since the flame stretch only depends on scales down to \( l_F \), scales close to the Kolmogorov scale do not dominate the flame stretch any more. Thus, as \( u'/s_L \) (or the flow Reynolds number) is increased, the ratio \( \gamma_K \) decreases. The opposite is true for fixed \( u'/s_L \) and increasing \( L/l_F \).

We can also define the total stretch normalized by the large-scale strain as

\[
\Gamma_K = \frac{\langle K \rangle}{\langle e \rangle/k},
\]

where \( k = 3/2 u'^2 \) is the density of turbulent kinetic energy (Fig. 10). This parametrization is seen to be essentially independent of \( u'/s_L \), except for very high values of \( L/l_F \). This is because the integrand as well as the limits of the integration in Eq. 22 only depend on \( L/l_F \) when \( \eta_K < l_F \) (this condition can be shown to be equivalent to \( L/l_F < (u'/s_L)^3 \)). The prefactor is essentially the large-scale strain \( u'/l_F \). Only for very high values of \( L/l_F \) for which \( \eta_K > l_F \), some dependence on \( u'/s_L \) can be observed.

### 4.2 The Flame Quenching

From Fig. 3 we know that eddies whose velocity \( v_r \) is large will be able to quench the flame. In term of \( \alpha \), this regime corresponds to values \( \alpha \) obeying \( \alpha < \alpha_1 \), where \( \alpha_1 \) is given in Appendix A. Let us now consider a snapshot of the flame and let us focus on a particular region of size \( r \) of that flame. That location will be experiencing quenching if it happens to coincide with the location of an eddy of size \( r \) which has a velocity such that \( \alpha < \alpha_1 \). The probability density of observing such an \( \alpha \) value will be given by \( P_r(\alpha) \), which is therefore also the probability density that the flame will be quenched by an eddy of size \( r \) of that particular \( \alpha \), as long as \( \alpha < \alpha_1 \). The probability that the flame is being quenched at that location is therefore

\[
\Pi_r(\tau_r) = \int_{-\infty}^{\alpha_1} P_r(\alpha) d\alpha.
\]

Again performing the integration over \( \alpha \) yields

\[
\Pi_r(\tau_r) = \frac{1}{\sqrt{\Pi}} \int_{-\infty}^{\beta_1} e^{-\beta^2} d\beta,
\]

where

\[
\beta_1 = \sqrt{\frac{\ln(L/r)}{2\mu}} \left( \alpha_1 - 3 - \frac{\mu}{2} \right).
\]

Therefore, when looking at a single snapshot of the flame, it would appear that quenching is occurring on a fraction \( \Pi_r(\tau_r) \) of the flame surface. However, if the eddy time-scale \( \tau_r \) is smaller than the flame time-scale \( t_q \), then such eddies will not be able to complete the quenching, since they must at least survive for a time equal to \( t_q \). We can estimate the probability that a succession of such eddies exist at the right location, so that quenching conditions exist for a time-span \( t_q \).
The situation is depicted in Fig. 11, where a temporal sequence of approximately \( t_q / \tau_r \) consecutive eddies of size \( r \) with \( \alpha < \alpha_1 \) must exist in order to induce actual quenching. We now want to estimate the probability of occurrence of such a sequence.

We know that for eddies of size \( r \) the probability that over a time-span \( \tau_r \) quenching conditions will exist is \( \Pi_r(\tau_r) \). After a turnover time \( \tau_r \) of the eddies, we assume that another, statistically independent configuration of \( r \)-eddies exists along the flame surface. Then the probability that at both stages the same portion of the surface be subjected to quenching is estimated as the product of the (independent) single-step probabilities, namely

\[
\Pi_r(2\tau_r) = \left[ \Pi_r(\tau_r) \right]^2. \tag{28}
\]

Then, the probability that a portion of the flame be subjected to quenching conditions for a time \( t_q \) (and therefore gets really quenched) is estimated as

\[
\Pi_r(t_q) = \left[ \Pi_r(\tau_r) \right]^{\beta_r}, \tag{29}
\]

where

\[
\beta_r = \frac{t_q}{\tau_r} \text{ if } t_q > \tau_r \tag{30}
\]

or

\[
\beta_r = 1 \text{ if } t_q < \tau_r. \tag{31}
\]

To compute the mean eddy time-scale \( \tau_r \), we use the estimate

\[
\tau_r \approx \frac{1}{\langle S_r \rangle} = \frac{L}{u' \left( \frac{r}{L} \right)^{2/3 - \alpha/9}}. \tag{32}
\]

To take into account all relevant eddy sizes, we proceed as follows: We consider a discrete set of scales \( r_n \) such that

\[
r_n = 2^{-n}L, \quad n = 1, 2, \ldots, n_{\text{max}}, \tag{33}
\]

where

\[
n_{\text{max}} = \text{int} \left[ \log_2 \left( \frac{L}{r_{\text{min}}} \right) \right]. \tag{34}
\]

and \( r_{\text{min}} \) is the minimum eddy-size, as in the preceding section. We start with the large eddies of the flow, \( r = L/2(n = 1) \) and compute \( \Pi_{r_{\text{max}}}(t_q) \). (We do not include \( r = L \) because strictly speaking Eq. 13 is valid only for \( r < L \).) The fraction of the flame not undergoing quenching due to eddies of size \( r_1 \) is thus \( \Pi_{r_{\text{max}}}^{NQ} = 1 - \Pi_{r_1}(t_q) \). Then we consider the next smaller scale, with \( n = 2 \) and compute \( \Pi_{r_{\text{max}}-2}(t_q) \). Now, the remaining unquenched fraction is \( \Pi_{r_2}^{NQ} = 1 - \Pi_{r_2}(t_q)\Pi_{r_{\text{max}}}^{NQ} \). At level \( n \), we have

\[
\Pi_{r_n}^{NQ} = \left[ 1 - \Pi_{r_n}(t_q) \right] \Pi_{r_{n-1}}^{NQ}, \tag{35}
\]

or

\[
\Pi_{r_n}^{NQ} = \left( 1 - \int_{-\infty}^{\alpha_1} P_{r_n}(\alpha) d\alpha \right)^{\beta_n} \Pi_{r_{n-1}}^{NQ}. \tag{36}
\]

Fig. 11. Schematic diagram of the quenching cascade: Eddies of size \( r \) must occur successively \( t_q / \tau_r \) times in order to be able to produce quenching.
where
\[ \beta_n = \min \left[ 1, \left( \frac{L}{L_{SL}} u' \right)^{2(3-\mu_/9)n} \right], \]  
(37)
and
\[ r_n = 2^{-n} L. \]
(38)

We refer to this model as the quenching cascade. Finally, the total fraction of surface undergoing quenching during a time \( t_q \) will be given by
\[ P_q = 1 - \Pi_{t_{min}}^{NQ} \]  
(39)
where \( \Pi_{t_{min}}^{NQ} \) can be computed recursively with Eq. 36, starting with \( n = 1 \). We point out that this calculation does not take into account some other plausible sequences of eddies that could also lead to quenching: The succession in time of eddies of different sizes at the same location. However, the results are not very sensitive to the addition of such effects (but it should be remembered that their omission might slightly underestimate quenching).

Figure 12 displays lines of constant \( P_q \), as a function of turbulence conditions. The transition from almost no quenching (\( P_q = 0 \)) to almost complete quenching (\( P_q = 1 \)) occurs in a relatively small strip of the parameter range, which increases with \( L / L_F \). As expected, for high turbulence levels, quenching conditions exist. Whether these conditions lead to a total quenching of the flame or to a regime of distributed reaction zones cannot be deduced from the present work.

### 4.3. The Net Flame Stretch

In the previous two sections, we have computed both the rate at which flame surface is created by effective stretch, and the fraction of flame surface that is quenched after a time \( t_q \). In the absence of quenching and consumption, the flame surface after a time \( t_q \) would be
\[ A(t_q) = A_0 e^{(K)t_q}, \]
(40)
where \( A_0 \) is the surface at \( t = 0 \). However, if after such a time a fraction \( P_q \) of the surface has been quenched, the net active flame surface will be
\[ A_N(t_q) = A(t_q)(1 - P_q) = A_0(1 - P_q)e^{(K)t_q}. \]
(41)
Let us now define the net flame stretch \( \overline{K} \) by
\[ A_N(t) = A_0 e^{\overline{K}t}, \]
(42)
where we require that \( A_N(t_q) \) be given by Eq. 41 at \( t = t_q \). The net stretch is the relevant quantity for flamelet models because it describes the growth of the surface where combustion actually takes place. It follows from Eq. 41 that \( \overline{K} \) is given by
\[ \overline{K} = (K) - \frac{1}{t_q} \ln \left( \frac{1}{1 - P_q} \right). \]
(43)

Therefore, the net stretch is smaller than the total flame stretch \( \langle K \rangle \) because a fraction of the flame surface which is created gets quenched, an effect that is not instantaneous but proceeds on a time scale \( t_q \). We can define a net stretch ratio \( \gamma_K \) by nondimensionalizing \( \overline{K} \) by the small-scale strain, according to
\[ \gamma_K = \frac{\overline{K}}{\left( \frac{\langle \epsilon \rangle}{\nu} \right)^{1/2}} = \gamma_K \left( \frac{L}{L_F} \right)^{1/2} \left( \frac{u'}{S_L} \right)^{-3/2} \ln \left( \frac{1}{1 - P_q} \right). \]
(44)

Figure 13 shows \( \gamma_K \) as a function of the flow turbulence parameters. For small turbulence in-
tensity levels, there is little quenching and the net stretch is a positive fraction of $\sqrt{\langle \epsilon \rangle \nu}$, increasing with $L/I_F$. When the turbulence intensity $u'/s_L$ increases, the net stretch becomes negative (net decrease in burning flame surface). At increasing $L/I_F$ and at fixed $u'/s_L$, vortices are larger but not faster so that the strain is smaller and the probability of quenching decreases leading to a positive net flame stretch again.

We can also nondimensionalize $\bar{K}$ by the large-scale strain $\langle \epsilon \rangle/k$:

$$
\Gamma_K = \frac{\bar{K}}{\langle \epsilon \rangle/k} = \Gamma_K - \frac{3}{2} \left( \frac{L}{I_F} \right) \left( \frac{u'}{s_L} \right)^{-1} \ln \left( \frac{1}{1 - P_q} \right),
$$

where $\Gamma_K$ is the (large-scale) stretch ratio of section 4.1. This is shown in Figure 14 as a function of the turbulence parameters $L/I_F$ and $u'/s_L$. In order to provide a useful tool for modeling, we propose a regression for computing $\Gamma_K$ as a function of the flow parameters ($L/I_F, u'/s_L$). The detailed form of the fit is presented in Appendix B.

4.4. Global Quenching Criteria

Complete flame quenching in turbulent combustion is a complex mechanism which is very often situation-dependent. However, the previous results may be used in simple ways to derive two types of quenching criteria. The first approach is described in Section 4.4.3 and gives an upper limit (in diagrams of premixed turbulent combustion) beyond which the turbulent flame will be quenched in any case because the net stretch is negative (Fig. 15). The second approach (Section 4.4.4) takes into account the interaction between the quenched and the active parts of the flame front (which leads to a dilution of the burned gases by fresh unburned gases) to derive a more realistic criterion in which the equivalence ratio plays an important role. Before describing these two approaches, we will recall how the classical criterion of Klimov-Williams [3] and the minimal criterion of Poinsot et al. [8] are obtained (Sections 4.4.1 and 4.4.2). Because all those criteria try to predict the limits of flamelet quenching in a turbulent flame, they also indicate the boundaries of the flamelet domain in the diagrams for premixed turbulent combustion [2, 3] as defined in Section 1.1. We will also present experimental correlations by Abdel-Gayed and Bradley [6, 7].

4.4.1. The Klimov-Williams Criterion for Quenching.

The first criterion for flamelet quenching was proposed by Klimov [2, 3]. This criterion implicitly assumes that the turbulent strain is completely converted into flame stretch (the efficiency function is supposed to be unity for all scales). Define the Karlovitz number $Ka$ at the ratio of the strain at the Kolmogorov scale $\nu_K/\eta_K$ to the critical stretch for flame extinction $s_L/I_F$. The Klimov Williams criterion states that flamelets will be quenched if $Ka$ is larger than unity. Note that many equivalent expressions for $Ka$ may be ob-
4.4.3. An Upper Limit for Quenching Given by the ITNFS Model.

From the estimate of the net flame stretch provided by the ITNFS model (Section 4.3), it becomes clear that whenever $K < 0$, the total flame area will tend to zero and the flame will extinguish or burn in a completely different manner (for example, in a distributed reaction regime). Therefore, the condition $K = 0$ defines an upper limit of flamelet quenching. No flamelet regime can be sustained above this limit. Using the condition $I'_{q} = 0$ in Eq. 45 gives the minimum value of $P_{q}$ producing quenching:

$$P_{q} \left( \frac{L}{l_{F}}, \frac{u' / s_{L}}{l_{F}} \right) = 1 - e^{-2/3 \Gamma_{k}(l_{F}/L)(u' / s_{L})},$$

where $\Gamma_{k}$ is the (large-scale) stretch ratio of Section 4.1. This is solved numerically for all parameters $(L / l_{F}, u' / s_{L})$, and the resulting limit is shown in Fig. 15.

4.4.4. Quenching Criterion Based on the Dilution of the Hot Gases.

In the previous section, we have assumed that the quenched portions of the flame do not influence the development of the active part of the flame. Such an assumption obviously breaks down after some time because fresh gases penetrate into the hot gases through the quenched part of the flame surface, dilute the burned gases and decrease their temperature $T_{2}$ [8]. If $T_{2}$ decreases too much, then, even in the absence of flame stretch, complete quenching will occur due to the large
subadiabaticity \( \Delta T = T_{ad} - T_2 \) of the stream of burned gases [2] \( T_{ad} \) is the adiabatic flame temperature). To first order, this effect may be estimated by assuming that the temperature \( T_2 \) of the burned gases in the turbulent flame brush is the result of the mixing process of the stream of burned gases at the adiabatic temperature \( T_{ad} \) produced by the active flame surface and of the stream of cold gases at temperature \( T_1 \) entering the hot gases through the quenched flame surface (Fig. 16). Although the flow rates associated to this mixing process are unknown, the subadiabaticity of the hot gases will be a monotonous increasing function of the fraction of quenched surface \( P_q \) (when the fraction of quenched flame surface increases, the subadiabaticity of the hot gases increases):

\[
\Delta T = G(P_q).
\] (48)

The degree of subadiabaticity \( \Delta T^{\text{crit}} \) which a flame can sustain before getting completely quenched is a function of its chemical parameters. To first order, it is given by a curve shown in Fig. 17. Close to the flammability limits, the subadiabaticity margin will be a monotonous increasing function of the fraction of quenched surface \( P_q \) (when the fraction of quenched flame surface increases, the subadiabaticity of the hot gases increases):

\[
\Delta T = G(P_q).
\] (48)

The degree of subadiabaticity \( \Delta T^{\text{crit}} \) which a flame can sustain before getting completely quenched is a function of its chemical parameters. To first order, it is given by a curve shown in Fig. 17. Close to the flammability limits, the subadiabaticity margin will be a monotonous increasing function of the fraction of quenched surface \( P_q \) (when the fraction of quenched flame surface increases, the subadiabaticity of the hot gases increases):

\[
\Delta T = G(P_q).
\] (48)

for the critical quenching probability \( P_q^{\text{crit}} \). This is done in Fig. 15 for a typical lean flame where \( P_q^{\text{crit}} \) would be, for example \( P_q^{\text{crit}} = 0.01 \). For a stoechiometric flame, \( P_q^{\text{crit}} \) would be higher (we took \( P_q^{\text{crit}} = 0.5 \)).

4.4.5. Comparison of the Quenching Criteria and Discussion.

Figure 15 presents all quenching criteria in the premixed turbulent combustion diagram. The line labeled (1) is the Klimov-Williams line, the lower limit of [8] is plotted in curve (2), the upper limit given by the ITNFS model is curve (3) and curves (4) and (5) correspond to the quenching limits based on the dilution of the hot gases with \( P_q^{\text{crit}} = 0.01 \) and \( P_q^{\text{crit}} = 0.5 \). The line labeled (6) is the experimental correlation of Abdel-Gayed and Bradley [6, 7] obtained from studies of quenching in fan-stirred combustion bombs.

All quenching limit curves (2) to (6) are located above the Klimov-Williams line (1). The Klimov-Williams criterion overestimates quenching by assuming that Kolmogorov scales can quench a flame. In fact, these scales are too small and are dissipated by viscosity too fast to affect the flame front [8].

The single-eddy lower limit (2) predicts quenching very closely to the \( P_q = 0.5 \) line (curve 4) because it assumes that one vortex with average strength (corresponding to the Kolmogorov speed at its particular size) is enough to quench the flame front. The upper quenching limit (3) suggested by the ITNFS model predicts very high values of turbulence before quenching.
is achieved. In fact, as indicated above, this is an absolute upper limit because interactions between active and quenched parts of the flame surface will probably lead to quenching before the net stretch rate (which neglects these interactions) becomes zero. Curves (4) and (5) take these interactions into account. It must be stressed that due to the stochastic character of the vortex speeds, even if one is below any of these quenching limits, there will always be some fraction of the flame that is being quenched locally (leading to a partially quenched regime), although this fraction becomes quite negligible as one moves toward low intensity turbulence (see Fig. 12).

Additional experimental studies are needed to settle the issue of which of these curves represents the most realistic quenching limit for real flames. A first obvious result, however, is that premixed flames are more difficult to quench than initially suggested by the Klimov-Williams criterion. This result confirms experimental studies of flame quenching in fan-stirred explosion vessels [6, 7] or of flame blowoff in stabilized flames [40]. A comparison was performed in [8, 9] between the lower quenching limit (2) and the experimental results of Abdel-Gayed et al [6, 7]. The lower limit (2) may be parametrized by

\[ u'/s_L > 4Re_L^{0.25} \]  

(50)

and

\[ Re_L = u'L/\nu > 250. \]

(51)

Equations (50) and (51) set minimum values for quenching on the turbulent RMS speed \( u' \) and on the turbulent Reynolds number \( Re_L \) respectively. In their study of quenching of premixed flames in fan-stirred bombs, Abdel-Gayed and Bradley [6, 7] indicate two different correlations for quenching data:

(1) for \( Re_L > 300 \), partial quenching occurs when \( u'/s_L > 2Re_L^{0.25} \) and total flame quenching for \( u'/s_L > 3Re_L^{0.25} \) (line labeled (6) in Fig. 15). While heat losses were certainly present in the experiment, their influence was not documented. Condition (50) gives the same functional dependence as Abdel-Gayed and Bradley’s results. Note that a criterion based on quenching by dilution of the hot gases (see curves 4 and 5 in Fig. 15) with a critical quenching probability \( P_q^{\text{crit}} = 0.25 \) would give a constant close to the experimental value. Taking into account the scatter of the experimental results, the agreement between the values of the proportionality coefficient (between 3 and 4 for direct simulation condition, 3 for experimental results) is satisfactory.

(2) for \( Re_L < 300 \), quenching was obtained in experiments but could not be correlated using a function similar to condition (50). The \( Re_L = 300 \) limit of Abdel-Gayed and Bradley corresponds very well to condition (51) \( (Re_L = 250) \) below which our computation indicates that stretch cannot quench a flame front. This suggests that quenching for \( Re_L < 300 \) is not due to stretch but more probably thermal processes.

5. CONCLUSIONS

We have used direct simulations of flame-vortex interactions and a model for intermittent turbulence to estimate the total rate of stretching as well as the surface-fraction of quenched flame surface in premixed turbulent combustion. Two types of results have been obtained:

(1) An expression has been derived for the net flame stretch, which is the rate of growth of the active flame surface. The net flame stretch is a function of two parameters: the ratio of the integral scale to the flame thickness \( \frac{L}{l_F} \) and the ratio of the rms turbulence velocity to the laminar flame speed \( \frac{u'}{s_L} \). This quantity may be used directly in flamelet models for premixed turbulent flames [12, 17, 15].

(2) Two quenching criteria taking into account the interaction between a flame front and a complete turbulent flow field have been derived. They were compared (1) with the classical Klimov-Williams criterion, (2) with a criterion proposed by Poinsot et al. [8, 9], who assumed that the presence of a single eddy able to locally quench the flame was sufficient to lead to global quenching, and (3) to experimental results of Abdel-Gayed and Bradley. Quenching is overestimated by the classical scaling argument of Klimov-Williams, when the prefactor is taken to be unity. The criterion of Poinsot et al. neglects intermittency as well as the fact that a flame front will be able to recover from the action of an isolated eddy. These two mechanisms are included in the present ITNFS approach. Our first estimate of the quenching limit based on the ITNFS model was derived by expressing that the
net flame stretch is zero. This yields an upper position of the quenching limit because no turbulent reacting flow featuring flamelets could keep on burning when the associated net flame stretch is negative. A second estimate of quenching based on ITNFS results was then obtained by taking into account the interaction between active and quenched flame surface. This criterion gives quenching limits which depend on the level of subadiabaticity which the flame can sustain before total extinction (and therefore on the equivalence ratio). These limits are close to the single-eddy criterion of Poinsot et al. [8] and to the experimental results of Abdel-Gayed and Bradley [6, 7]. It is clear that those results need to be confirmed by experiments. They suggest a certain number of fundamental mechanisms which call for verification. However, it must be recalled that quenching is a strong function of heat losses [8] and that heat losses are difficult to estimate in experiments. Therefore, as shown in this study, numerical computations of flame quenching could be an adequate tool to investigate this phenomenon.

A general result independent of the previous limitations is that flamelet quenching in premixed turbulent combustion requires very high turbulence intensities and that the flamelet assumption is valid in a domain which is larger than expected from classical theories [2, 3].

This study was supported by the Center for Turbulence Research. The help of Dr Sanjiva Lele in developing the direct simulation code is gratefully acknowledged. We also thank Dr. Arnauld Trouvé for many discussions and his comments on the manuscript.

APPENDIX A

This appendix deals with the limits of integration for the integrals of section 4.2. From Fig. 3, we fit a function $g(s)$ to the quenching limit, so that

$$\left(\frac{v_r}{s_L}\right)_{\text{max}} = 10 g(\log_{10}(r/l_F)). \quad (52)$$

The function $g$ is chosen according to the following requirements:

$$\lim_{s \to \infty} g(s) = s, \quad \lim_{s \to -1/2} g(s) = (s + 1/2)^{-1}. \quad (53)$$

The first condition follows from $Ka(r) = 1$ and the second from the observation that scales smaller than $l_F/3$ appear not to quench the flame (see Fig. 3). A smooth crossover with exponential matching that loosely fits the results of Poinsot et al. [8] is given by

$$g(s) = 1.3 + 1.667 \frac{1}{s + 1/2} e^{-1.2(s+1/2)} + (1 - e^{-1.2(s+1/2)}) (s).$$

Combining Eqs. 11 and 12, using $\langle \epsilon \rangle = u'^3/L$ and neglecting prefactors of order unity, we can write

$$\alpha(x) = 2 + \frac{\ln \left(\frac{r}{L}\right)}{\ln \left(\frac{u'}{s_L}\right)\ln(10)}.$$ \quad (54)

The upper limit for $v_r/s_L$ thus gives rise to the following limit of $\alpha$:

$$\alpha_1 = 2 + \frac{3g \left(\log_{10}\left(\frac{r}{L}\frac{L}{l_F}\right)\right)\ln(10) - 3 \ln \left(\frac{u'}{s_L}\right)}{\ln \left(\frac{r}{L}\right)}.$$ \quad (55)

Therefore, the limit of integration over $\alpha$ depends on $p$, as well as on the "global" (large-scale) properties of the flame $u'/s_L$ and $l_F/L$.

APPENDIX B

The complete computation program may be obtained directly from the authors. However, fits are easier to use in a model. In this appendix we present possible fits to obtain the net stretch ratio for different turbulence and flame parameters. Our starting point is the definition of the net stretch ratio.

$$\Gamma_K \left(\frac{L}{l_F}\frac{u'}{s_L}\right) = \Gamma_K - \frac{3}{2} \left(\frac{L}{l_F}\right) \left(\frac{u'}{s_L}\right)^{-1} \times \ln \left(\frac{1}{1 - P_q}\right). \quad (56)$$
First we fit the stretch ratio $\Gamma_K$. From Fig. 10, we see that only a weak dependence on $u'/s_L$ is required. We use a $2/3$ power law for high $L/l_F$ values (this exponent is decreased somewhat at low $u'/s_L$). For values of $L/l_F$ tending to 0.4, we require that the stretch go to zero very quickly. We use exponential matching between both limiting behavior

$$
\Gamma_K = 10^{r(s, u'/s_L)},
$$

where

$$
r(s, u'/s_L) = - \frac{1}{(s + 0.4)} e^{-(s+0.4)} + (1 - e^{-(s+0.4)}) \left( \sigma \left( \frac{u'}{s_L} \right) s - 0.11 \right),
$$

$$
s = \log_{10} \left( \frac{L}{l_F} \right),
$$

and

$$
\sigma \left( \frac{u'}{s_L} \right) = \frac{2}{3} \left( 1 - \frac{1}{2} e^{(u'/s_L)^{1/3}} \right),
$$

To fit $P_q(L/l_F, u'/s_L)$, we use the function $\tanh(x^2)$ centered around the line $\log_{10}(u'/s_L) = g(L/l_F)$ where $P_q = 0.5$. The width of the argument is given by another function $\sigma(L/l_F)$, which increases with $L/l_F$. A reasonably good fit results when using

$$
P_q \left( \frac{L}{l_F}, \frac{u'}{s_L} \right) = \frac{1}{2} \left[ 1 + \tanh(\text{sgn}[x] x^2) \right],
$$

where

$$
\log_{10} \left( \frac{u'}{s_L} \right) - g \left( \frac{L}{l_F} \right)
$$

$$
\sigma \left( \frac{L}{l_F} \right)
$$

Combining these results allows for the calculation of the net-stretch ratio $\Gamma_K$, which can then be used in any flamelet model for premixed turbulent combustion. Note that the importance of quenching may be diminished by decreasing the quenching term in Eq. 56. In the limit of perfectly adiabatic flames, only the first term $\Gamma_K$ should be used.

REFERENCES

332 C. MENEVEAU AND T. POINSOT


Received 19 July 1990; revised 28 February 1991