Distribution of Energy Spectra, Reynolds Stresses, Turbulence Production, and Dissipation in a Tidally Driven Bottom Boundary Layer

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ABSTRACT

Seven sets of 2D particle image velocimetry data obtained in the bottom boundary layer of the coastal ocean along the South Carolina and Georgia coast [at the South Atlantic Bight Synoptic Offshore Observational Network (SABSOON) site] are examined, covering the accelerating and decelerating phases of a single tidal cycle at several heights above the seabed. Additional datasets from a previous deployment are also included in the analysis. The mean velocity profiles are logarithmic, and the vertical distribution of Reynolds stresses normalized by the square of the free stream velocity collapse well for data obtained at the same elevation but at different phases of the tidal cycle. The magnitudes of $u'^2$, $w'^2$, and $u'w'$ decrease with height above bottom in the 25–160-cm elevation range and are consistent with the magnitudes and trends observed in laboratory turbulent boundary layers. If a constant stress layer exists, it is located below 25-cm elevation. Two methods for estimating dissipation rate are compared. The first, a direct estimate, is based on the measured in-plane instantaneous velocity gradients. The second method is based on fitting the resolved part of the dissipation spectrum to the universal dissipation spectrum available in Gargett et al. Being undervalued, the direct estimates are a factor of 2–2.5 smaller than the spectrum-based estimates. Taylor microscale Reynolds numbers for the present analysis range from 24 to 665. Anisotropy is present at all resolved scales. At the transition between inertial and dissipation range the longitudinal spectra exhibit a flatter than $-5/3$ slope and form spectral bumps. Second-order statistics of the velocity gradients show a tendency toward isotropy with increasing Reynolds number. Dissipation exceeds production at all measurement heights, but the difference varies with elevation. Close to the bottom, the production is 40%–70% of the dissipation, but it decreases to 10%–30% for elevations greater than 80 cm.

1. Introduction

Understanding and proper representation of coastal bottom boundary layer processes, either of biological or physical nature, require knowledge of the flow dynamics including turbulence generation and transport. The bottom boundary layer is the zone where exchanges of chemicals and organisms (e.g., larvae) between the seabed and the overlying water column take place. Lentz (1995) points out that the knowledge of turbulent stresses throughout the water column, including at the bottom, play a major role in the flow dynamics and in the overall shelf circulation. The bottom boundary layer is usually not resolved in circulation models, and parameterizations of turbulence and bed stress are based upon assumptions about stress profiles that originate from atmospheric or laboratory flows.

The water motion in the coastal waters and associated bottom boundary layer is driven by a number of mechanisms including winds, tides, density gradients, swells, sea surface slope and is affected by the Coriolis force (Grant and Madsen 1986). The relative importance of the various mechanisms varies from one region to another and depends on the time and spatial scales of the motion one wishes to resolve. In this study we investigate the structure and scaling of turbulence parameters in the boundary layer on the continental shelf using data obtained from in situ measurements with a submersible particle image velocimetry (PIV) system.

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We start with a brief theoretical and experimental background. Measurements of turbulent stresses in the bottom boundary layer have traditionally been indirect, and relied upon various assumptions. The “profile method” (Grant and Madsen 1986) depends on existence of the logarithmic layer and uses the law of the wall. The inertial dissipation technique (Grant et al. 1984; Gross and Nowell 1985; Huntley and Hazen 1988) is based on the assumed balance of turbulence dissipation and production rates as well as existence of a logarithmic layer. The dissipation rate is estimated from a fit of the Kolmogorov $-5/3$ spectral slope in the inertial range of the vertical velocity spectrum. The dissipation technique (Dewey and Crawford 1988; Agrawal and Aubrey 1992) also makes the same assumptions, but utilizes velocity measurements at sufficiently small scales to obtain the dissipation rate from a fit to the universal spectrum in the dissipation range. Issues and difficulties are discussed in Gross et al. (1994). All of these methods assume existence of a constant stress layer; that is, the measured shear stress is assumed to be equal to the bottom stress. The dissipation estimates inherently include assumptions of isotropy and universality (Tennekes and Lumley 1972).

The eddy correlation technique, involving direct Reynolds stress measurements, is recognized to be very difficult to perform in the coastal ocean because of a variety of technical challenges such as tilt of the sensors, contamination by wave motions, and lack of spatial resolution (Grant and Madsen 1986). Trowbridge (1998) and Shaw and Trowbridge (2001) resolve the wave contamination and instrument tilt problem by estimating the stress from the second-order structure function of velocity components obtained using two spatially separated sensors. Nimmo Smith et al. (2002) and Osborn et al. (2007, manuscript submitted to J. Phys. Oceanogr., hereinafter referred to as O07) utilize this approach using PIV data, and the latter also provide expressions for wave contamination for each stress component.

Over the last 50 years a significant body of knowledge has been accumulated on spectral properties of turbulence (Pope 2000). Some of the landmark articles that are relevant to the present paper are Grant et al. (1962), Champagne et al. (1977), Champagne (1978), Gargett et al. (1984), and Saddoughi and Veeravalli (1994). Others are mentioned in a review paper by Gargett (1989). These studies have been motivated by Kolmogorov’s theories (Pope 2000) that at sufficiently high Reynolds numbers, the turbulence has a universal spectrum of the form

$$E(k) = (\epsilon r^5)^{1/4} \Phi(k \eta),$$  

(1)

where $E(k)$ is the three-dimensional spectral density function, $\epsilon$ is the viscous dissipation rate, $r$ is the kinematic viscosity, $k$ is the wavenumber magnitude, $\eta = (\nu^2/\epsilon)^{1/4}$ is the Kolmogorov length scale, and $\Phi$ is a universal function valid for all turbulent velocity fields in their equilibrium range of wavenumbers. Within the spectrum there is a viscosity independent, inertial subrange for which

$$E(k) = C \epsilon^{2/3} k^{-5/3},$$  

(2)

where $C$ is a universal constant. An essential part of these theories is the assumption of isotropy, which has been extended to “local isotropy”; that is, at sufficiently large Reynolds number, the small-scale turbulent structure is isotropic even when the large-scale structure is not. Existence of an inertial range in oceanic turbulence has been identified by Grant et al. (1962) and Gargett et al. (1984, henceforth, this latter paper is referred to as G84), both away from boundaries. A spectral range with $-5/3$ slope in vertical velocity component spectra has also been observed and utilized in bottom boundary layer analyses by Grant et al. (1984) and Gross and Nowell (1985).

In all of the experimental work cited above, Taylor’s hypothesis is used to deduce wavenumber spectra from frequency spectra of a single sensor. Applicability of Taylor’s hypothesis relies on the existence of statistically stationary flow conditions, which are essential for obtaining the advective velocity. Conversely, PIV measures the instantaneous distributions of two velocity components within a sample plane. Thus, the spatial spectra can be calculated without using the Taylor’s hypothesis. Comparisons between the PIV spatial spectra and corresponding temporal spectra based on Taylor’s hypothesis have been examined by Bertuccioli et al. (1999) and Doron et al. (2001). They show that outside of the range where waves affect the temporal spectra, there is a little difference between them. Some of the spatial spectra of previous ocean measurements, especially those obtained at low mean currents near the bottom, deviate substantially from the universal profile and include considerable anisotropy (Doron et al. 2001; Nimmo Smith et al. 2005, hereinafter referred to as NS05). The extent and implication of this anisotropy are discussed later in this paper. Section 2 describes the measurements, submersible PIV system, and the flow conditions encountered during the measurements. The mean flow properties, turbulent stresses, dissipation and production rate estimates, and spectral analysis are presented in section 3, while section 4 covers the main conclusions of the paper. Overall, we show that the measurements when properly scaled are consistent with
the laboratory results from turbulent boundary layers, although data show substantial variability in trends of Reynolds stresses, production–dissipation balance, and spectral shapes that depend on elevation, Reynolds number, and phase within the tidal cycle.

2. Experimental details

a. Submersible PIV system

The PIV system, illustrated in Fig. 1a, is the second-generation design from the original rig described in Bertuccioli et al. (1999) and Doron et al. (2001). A detailed description of the present system can be found in Nimmo Smith et al. (2004). Briefly, the submersible PIV system consists of two principal components, one located on the ship, and the other submerged in the water. The shipboard component (not shown here) consists of a laser, associated optics, acquisition and control computers, and high-speed disk arrays for data storage. The light source is a dual-head, dye laser with pulse duration of 2 μs and maximum output of 350 mJ per pulse at 594-nm wavelength. The laser beam is split and transmitted through two 60-m-long optical fibers to the submerged part, which contains two independent probes with laser sheet forming optics. The resulting light sheet thickness is about 3 mm at the center of the sample area and slightly diverges to 4 mm at the edges. The images are recorded using two CCD cameras each with $2048 \times 2048$ pixels resolution and are equipped with electronic image shifters that resolve the directional ambiguity problem while recording two-exposure
images. For measurements described in this paper, the cameras are located 120 cm from the light sheet, have sample areas of $35 \times 35$ cm$^2$ (in water) each, and are separated horizontally by 31 cm (see Fig. 1b). The time delay between the two laser pulses is adjusted according to the mean streamwise velocity, and in most cases is 8 ms.

The cameras, light sheet probes, and the suite of support environmental sensors are rigidly mounted on a profiling platform that has an extension capability of 10 m. The entire system is mounted on a turntable that can be rotated to align the illuminated sample areas with the mean flow. The mean flow direction is determined using a vane monitored by a video camera. Other environmental sensors include precision pressure sensor, CTD, as well as compass and pitch and roll sensors located in each of the camera housings. In addition, for this experiment the system is equipped with a microscopic camera whose primary aim is to sample the size, shape, and distribution of particles close to the sample area. All instruments on the submersible system are operated simultaneously with the PIV measurements.

b. Analysis of PIV data

The acquired images are analyzed using a correlation analysis code developed in-house (Roth et al. 1999; Roth and Katz 2001; Nimmo Smith et al. 2004). The interrogation windows size is $64 \times 64$ pixels, resulting in the interrogation volume of $1.09 \times 1.09 \times 0.3$ cm$^3$ for each vector. With 50% overlap between windows the data yields $63 \times 63$, 2D instantaneous velocity vectors per image. The typical uncertainty in instantaneous velocity is about 2%. The seed particles of the present measurements are naturally suspended particles of predominantly biological material. Microscopic images of the particles taken close to the PIV sample areas show that more than 90% of the particles are smaller than 50 µm in diameter (Fricova 2005). Since the seed particles are not distributed uniformly within the sample volume, instantaneous vector fields that contain less than 70% of the total number of vectors are not used in subsequent analysis. For the present datasets, very few (0.1%) of the images have to be discarded. For calculating the dissipation rate estimates, gaps within the velocity vectors are filled by linear interpolation as explained in NS05, and two outermost rows and columns of data in each vector map are truncated because of lower data quality near the edges, resulting in $59 \times 59$ vectors per instantaneous realization. Because of the large field of view, the PIV data are calibrated to compensate for the variations in magnification across the sample area. Images of a target (recorded prior to the deployment), whose surface contains several hundred precisely and evenly spaced markings, are used for matching the sample-area “fluid” plane with the camera’s sensor “image” plane (Soloff et al. 1997; Nimmo Smith et al. 2002).

c. Site, flow conditions, and data acquired

The PIV system was deployed from the Research Vessel (R/V) _Cape Hatteras_. The cruise took place from 6 to 20 June 2003, in the vicinity of the South Atlantic Bight Synoptic Observational Network (SABSOON), located on the continental shelf along the South Carolina and Georgia coast. SABSOON consists of eight instrumented towers covering an area of $115 \times 50$ km predominantly in the alongshore direction. The series of measurements described in this paper were performed at 31.34°N, 80.69°W, about 10 n mi south-southwest from the innermost tower designated R2. The mean water depth at this location was 23 m, the seabed consisted of sand and broken shell, and bottom survey using the onboard echo sounder indicated a bottom slope of 3/700. The mean depth-averaged current speed, as measured by the onboard acoustic Doppler current profiler (ADCP), an RD Instruments, Inc., 150-kHz deep water sensor set to average data over 1-m-depth bins, is presented in Fig. 2. The depth-averaged speed is obtained using 11 depth bins starting 9.5 m from the sea surface, to a depth of 19.5 m (see also Fig. 3c), that is, 3.5 m above bottom. Figure 2 indicates that strong semi-diurnal tidal currents dominate the mean flow with magnitude ranging from 10 cm s$^{-1}$ up to about 50 cm s$^{-1}$. As indicated by the plus signs, four PIV datasets are obtained during the accelerating phase (runs 101 to 104) and three sets (runs 105 to 107) in the...
decelerating phase of the tidal cycle. Each run lasts for 20 min and the data are acquired at a constant rate of three frames per second, resulting in 3600 PIV realizations per run. The experiment concentrated on the bottom boundary layer starting from 0.43 m at the center of the sample area (the boundaries of the lowest sample area are 0.43 m ± 0.175 m) up to 2.5 m above the bed. The elevations of the center of the sample area in each of the seven runs together with the starting and ending depth-averaged current magnitudes are given in Table 1. The heights above bottom (wall-normal coordinates) are determined based on the pressure readings, after being corrected for variations in water elevation during the tidal cycle.

In between PIV measurements we performed three CTD casts marked by circles in Fig. 2. Cast 1 was taken between runs 101 and 102, cast 2 between runs 104 and 105, and cast 3 between runs 105 and 106. Individual profiles along with the corresponding ADCP current distributions are shown in Fig. 3. Temperature measurements with our system-mounted CTD during runs 102, 104, 105, and 106 are also presented in Fig. 3a. This CTD collects data for the full duration of PIV measurements, providing 20-min temperature time history at each elevation. During the deployment, the vertical gradients in temperature and salinity indicate stable stratification, with a thermocline situated about 14 m above bed, well above the PIV measurement locations. Over the phase of the tidal cycle, temperature is constantly decreasing with time, while salinity increases. Temperature from the system-mounted CTD shows water temperature decrease of 0.2°C over the course of seven runs within the 2.5 m above bottom. This trend is consistent with temperature and salinity data obtained from the nearby SABSOON R2 leg that indicate temperature and salinity cycle on a semidiurnal frequency at this location. ADCP profile 1 (Fig. 3c) shows current diminishing toward the bottom, profile 2 indicates
mean shear just below thermocline, and profile 3 exhibits substantial shear for almost the entire water column. This existence of shear above the bottom boundary layer presents difficulty in identifying its upper boundary, and defining a length scale for the bottom boundary layer, as discussed later.

To complete the picture of the flow conditions, spectra of our precision pressure transducer (not shown) indicate a peak wave frequency at 0.13 Hz; that is, a period in agreement with those of wave buoy 41008, located about 5 nmi NW of the measurement site. Their significant height is 0.65 m, in agreement with our pressure signal and also with data from the SABSOON R2 leg.

3. Results

a. Mean velocity profiles

In the analysis that follows, the $x$, $y$, and $z$ coordinates, with the corresponding $u$, $v$, and $w$ instantaneous velocity components denote the streamwise, spanwise, and wall-normal directions in the frame of reference of the PIV system. For convenience, $U$, $V$, and $W$ denote ensemble average velocity components, $U_i = \langle u_i \rangle$. To represent ensemble averaging operation we use $\langle \rangle$ over 20-min runs (3600 realizations). Data spatially averaged over an entire vector map is denoted as $\langle \rangle_{\text{map}}$. Since we align the system with the mean flow prior to each run, Table 1 also presents the alignment of the PIV system with the current direction measured by the ships ADCP (shown also in Fig. 2). The vertical distributions of $U(z)$, is presented in Fig. 4. In addition to ensemble averaging, each curve is averaged over the horizontal direction using data from camera A. These mean profiles are normalized by the corresponding magnitudes of the depth and time-averaged (20 min) speed obtained from the ADCP, whose values are presented in Table 1. The mean velocity increases with height, but the datasets do not collapse onto a single curve. At each elevation we show two profiles obtained at different phases of the tidal cycle. Profiles 101–104, obtained

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**Table 1.** Depth-averaged current speed, heading, and alignment angle of the PIV system for each dataset. Both compass and ADCP headings are with respect to the magnetic north. Mean $U_{ADCP}$ is used in the subsequent analysis as a normalizing parameter.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Mean elev (m)</th>
<th>$U_{ADCP}$ at start (cm s$^{-1}$)</th>
<th>$U_{ADCP}$ at end (cm s$^{-1}$)</th>
<th>Mean $U_{ADCP}$ (cm s$^{-1}$)</th>
<th>PIV heading (°)</th>
<th>ADCP current heading (°)</th>
<th>Alignment angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>101</td>
<td>0.43</td>
<td>23.2</td>
<td>27.6</td>
<td>25.4</td>
<td>238.1</td>
<td>236.6</td>
<td>1.5</td>
</tr>
<tr>
<td>102</td>
<td>0.90</td>
<td>30.1</td>
<td>34.5</td>
<td>32.3</td>
<td>244.4</td>
<td>250.8</td>
<td>-6.4</td>
</tr>
<tr>
<td>103</td>
<td>1.51</td>
<td>37.0</td>
<td>40.6</td>
<td>38.8</td>
<td>256.3</td>
<td>252.9</td>
<td>3.4</td>
</tr>
<tr>
<td>104</td>
<td>2.49</td>
<td>43.6</td>
<td>47.8</td>
<td>45.7</td>
<td>255.1</td>
<td>255.8</td>
<td>-0.7</td>
</tr>
<tr>
<td>105</td>
<td>0.43</td>
<td>46.0</td>
<td>42.5</td>
<td>44.3</td>
<td>260.5</td>
<td>263.8</td>
<td>-3.3</td>
</tr>
<tr>
<td>106</td>
<td>0.86</td>
<td>39.5</td>
<td>37.8</td>
<td>38.7</td>
<td>241.3</td>
<td>266.0</td>
<td>-24.7</td>
</tr>
<tr>
<td>107</td>
<td>1.41</td>
<td>34.7</td>
<td>28.7</td>
<td>31.7</td>
<td>239.9</td>
<td>275</td>
<td>-35.9</td>
</tr>
</tbody>
</table>

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**Fig. 4.** Normalized mean streamwise velocity profiles measured during accelerating (runs 101–104) and decelerating (runs 105–107) phases of a tidal cycle: (a) linear plot, and (b) semilog plot. Note that the $y$-$x$ axes in (b) are reversed from those in (a). Values of $U_{ADCP}$ are given in Table 1.
during the accelerating phase of the tidal flow, seem to be continuous, and so are runs 105–107, which are obtained during the decelerating phase. However, an initial inclination to relate the profile to the sign of acceleration would be misleading because of possible variation in bottom roughness with orientation and because of misalignment between the PIV plane and the mean flow. Correction of these profiles to account for misalignment of the PIV measuring plane does not explain the asymmetry between accelerating and decelerating velocity profiles. As shown in Table 1, for runs 101–105 the relative angle between the PIV plane and the flow direction is less than 7°. For run 106 where the misalignment angle is larger (25°), the corrected velocity profile (not shown) appears to be reasonably close to that of run 102. For run 107, the large misalignment angle (36°) shifts the corrected velocity significantly beyond that of run 103. However, estimates of current heading are based on the ADCP depth-averaged data only, and the relative angle between the PIV measuring plane and the flow direction is less than 7°. Change in tidal direction starts at the bottom, and during run 107, the current heading at an elevation of 3.5 m is 5° lower than the depth-averaged value. Thus, it is likely that at \( z = 1.41 \text{ m}\) of run 107, the current misalignment is even lower. Since we are uncertain about the current directions close to the bed, particularly during run 107, we present uncorrected profiles for all the runs.

The semilog plots (Fig. 4b) show that individual velocity distributions have logarithmic profiles (discussed later), but the slopes differ. Values of \( U_{map} \) and \( W_{map} \) and their spatial gradients are given in Table 2 for both cameras. We also include the RMS values, \( (u_{map})_{RMS} \) and \( (w_{map})_{RMS} \), of the spatially averaged instantaneous velocity components, as representatives for the amplitude of wave-induced motion. The values of \( U_{map} \) of cameras A and B differ in magnitudes by 1%–3% but not in trends, presumably because of small differences in the site of measurement. The values of \( W_{map} \) are small, but in some cases, especially away from the bottom, they reach 3%–5% of the streamwise velocity. However, recall that the system orientation is adjusted between runs; thus variations in the \( W_{map}/U_{map} \) ratio may also be caused by bottom topography.

Vertical gradients of streamwise velocity are larger than all the other gradients, while \( \langle \partial W/\partial z \rangle_{map} \) has the lowest magnitudes. The mean shear \( \langle \partial U/\partial z \rangle_{map} \) decreases with the distance from the bed, but at the highest elevation does not vanish. Consequently, we cannot identify the upper boundary of the bottom boundary layer and do not have a length scale for normalizing the elevation. For this reason, the elevation axis of Fig. 4 is left dimensional. The values of \( \langle \partial U/\partial z \rangle_{map} \) and \( \langle \partial W/\partial z \rangle_{map} \) are comparable in magnitude and have opposite sign, consistent with continuity for a two-dimensional mean flow.

b. Profiles of turbulence stresses

Following Trowbridge (1998), Shaw and Trowbridge (2001), and O07, the instantaneous velocity \( u_i \) is decomposed into

\[
  u_i = U_i + u'_i + u'_w,
\]

where \( U_i = \langle u_i \rangle \), \( u'_i \) is the velocity fluctuation, and \( u'_w \) is the wave-induced motion. Because of effect of wave contamination, the in-plane Reynolds stresses \( \langle u'u' \rangle \), \( \langle w'w' \rangle \), and \( -\langle u'w' \rangle \) are estimated using the second-order structure function, \( D_{ij}(r, z) \), defined as

\[
  D_{ij}(r, z) = \sum_{k=1}^{n} \left[ u_i(x_k, z) - U_i(x_k, z) \right] \left[ u_j(x_k + r, z) - U_j(x_k + r, z) \right] \times \left[ u_i(x_k, z) - U_i(x_k, z) \right] \left[ u_j(x_k + r, z) - U_j(x_k + r, z) \right],
\]
where \( r \) is the horizontal separation distance between two points, and \( k \) indicates a specific coordinate in the vector map. The maximum number of samples for a fixed \( r \), denoted by \( n \), changes with the separation distance. If \( r \) is larger than the integral scale of the turbulence, and much smaller than the wavelength of surface waves, \( D_0(r, z) \approx 2(u/u') \). Uncertainty from wave contamination is discussed in Shaw and Trowbridge (2001) and O07. The PIV data enable us to calculate the distribution of \( D_0(r, z) \) as a function of the separation distance without a priori knowledge of the integral scale of turbulence, provided that the sample areas are large enough. It is for this purpose that we use the two-camera configuration shown in Fig. 1b, which enables us to calculate \( D_0(r, z) \) for \( 0 < r < 100 \) cm. To increase the sample size for a certain elevation we average the values of \( D_0(r, z) \) over seven neighboring rows, that is, 3.8-cm-wide strips. Thus, each distribution at a given elevation is an average of \( 7 \times 3600 = 25,000 \) samples, as well as multiple contributions at a fixed \( r \). Each dataset provides seven distributions of \( D_0(r, z) \) by dividing the sample areas into seven adjacent horizontal layers.

To demonstrate the structure function approach in estimating Reynolds stresses, we present sample distributions of \( \frac{1}{2}D_0(r, z) \) at two elevations. Figures 5a and 5b show distributions of \( \frac{1}{2}D_0(r, z) = 0.43 \) m and \( \frac{1}{2}D_0(r, z) = 1.51 \) m, corresponding to runs 105 and 103, respectively. Each figure contains three different distributions, two of them using only one sample area (a separation distance \( 0 < r < 35 \) cm) and the third covering both sample areas ("across cameras"—with \( 35 < r < 100 \) cm). In Fig. 5a, the structure functions for the two single sample areas are in good agreement. The structure functions calculated over both sample areas display asymptotic convergence for \( D_{13}(r, z) \) and \( D_{33}(r, z) \), while the distribution of \( D_{11}(r, z) \) is not fully converged even after 1 m. These results indicate that \(-\langle u'u' \rangle \) is 2 cm\(^2\) s\(^{-2}\), \langle u'u' \rangle is slightly larger than 8 cm\(^2\) s\(^{-2}\), and \langle w'w' \rangle is 4.5 cm\(^2\) s\(^{-2}\). The convergence of \langle w'w' \rangle and conversely, lack of convergence of \langle u'u' \rangle, is consistent with the characteristic existence of larger turbulence scales in the streamwise direction relative to the vertical scales. Also note that the distribution of \( D_{13}(r, z) \) is continuous across the transition from one to two cameras, whereas the distribution of \( D_{11}(r, z) \) and \( D_{33}(r, z) \) are not. As discussed in O07, this difference is possibly caused by wave contamination, which is higher in the distribution of \( D_{11}(r, z) \) and \( D_{33}(r, z) \) than \( D_{13}(r, z) \).

For run 103 (Fig. 5b) the structure functions do not exhibit asymptotic behavior within the separation distances afforded by the present PIV set up (~1 m). Asymptotic convergence of the structure functions at this \( z = 1.51 \) m and higher elevations is expected to occur at larger separations, presumably because of the effect of the distance from bottom on the turbulence scales. A larger separation between the two sample areas would improve the stress measurements. Consequently, for runs 103 and 107 (mean elevations 1.51 and 1.41 m, respectively) we estimate the stresses based on the values of \( \frac{1}{2}D_0(r, z) \) at the largest separation of \( r = 1 \) m. This approximation biases the stress estimates toward low values. We estimate this bias in two ways. First include least squares fitting of exponentially decaying functions to the \( D_0(r, z) \) plots and we look for the asymptotic value at larger separation distances. The second approach utilizes Grant (1958) normalized correlation functions to estimate stress reduction as a function of separation in the \( D_{11} \) and \( D_{33} \) structure functions. Both analyses suggest that we underestimate \( \langle u'u' \rangle \) by 10%, \langle w'w' \rangle by 15%, and \(-\langle u'u' \rangle \) by 35% for measurements performed at \( z > 1 \) m. For run 104 (\( z = 2.49 \) m), not shown here, the lack of asymptotic behavior reaches a level that prohibits us from using \( D_0(r, z) \) for estimating stresses. Thus, we do not present stress results for this run. In addition to the errors associated with structure function approach, stress estimates for runs 106 and 107 are underestimated because of misalignment of the PIV plane with respect to the mean flow. We estimate this error by rotating the Reynolds stress tensor about the \( z \) axis by 25° for both datasets. The results show 5% and 10% increase in \( \langle u'u' \rangle \) and \(-\langle u'u' \rangle \).

Distributions of Reynolds stresses with height above bottom are presented in Fig. 6a. Appropriate error bars due to the bias in stress estimates, as discussed above, are also shown. When the stresses are normalized with \( U_{ADCP}^2 \), the profiles below 1 m height collapse but there are some differences in \( \langle u'u' \rangle \) and \( \langle w'w' \rangle \) at higher elevations. Note that for runs 101 and 105 the agreement in normalized values involves substantial differences in dimensional values. All the stress components decrease in magnitude with increase in elevation within our range of measurements. This trend is particularly noticeable for the normalized shear stress, which exhibits substantial vertical gradient for runs close to the bed. Clearly, if a constant shear stress layer exists, as one would expect for boundary layers with small streamwise pressure gradient, it is located within 25 cm from the bottom.

Trends of normalized stresses and the actual values are consistent with laboratory and numerical data for turbulent boundary layers (Schlichting 1979; Saddoughi and Verravalli 1994, hereinafter referred to as SV94; Spalart 1988). A quantitative comparison with the data
of SV94 suggests that our present measurements are performed in the 40%–70% range of the total boundary layer thickness, δ. Thus, our measurements are performed outside of the inner layer, and as a result, scaling with outer boundary layer parameters should be expected, consistent with the collapse of the Reynolds stresses when scaled with $U_{ADCP}^2$. We cannot estimate the out-of-plane stresses from the available two-dimensional PIV data. However, we can infer the missing normal component, $\langle u'v' \rangle$ from data provided in SV94 at $y/\delta \approx 0.4$. Based on their result, $\langle u'v' \rangle \approx 0.4(\langle u'u' \rangle + \langle w'w' \rangle)$.

Figure 6b shows components of the 2D surrogate of the anisotropy tensor, defined as $b^{2D}_{ij} = \langle u'_i u'_j \rangle / \langle u'_i u'_k \rangle - (1/2) \delta_{ij}$, where $\delta_{ij}$ is the Kronecker delta. The 3D version of this tensor is regularly used to examine large-
scale anisotropy (Pope 2000). All components of the anisotropy tensor collapse for runs conducted at the same elevation, but at different times within the tidal cycle, indicating consistency and repeatability of trends. The values of \( b_{11}^{2D} = -b_{33}^{2D} = (u' u') - (w' w')/ [2((u' u') + (w' w'))] \) decrease with elevation, becoming essentially zero above 1.3 m; that is, there is large-scale anisotropy close to the wall, as expected for boundary layers. The shear component of the anisotropy tensor \( b_{13}^{2D} \) is nonzero at all elevations and indicates lack of large-scale isotropy. Since it does not vanish in comparison with \( b_{11}^{2D} \) at our highest measurement location,
the distributions of shear stress also does not help in identifying the outer edge of the bottom boundary layer. As will be shown in the next section, the turbulence is also anisotropic at small scales.

Utilizing the above approximation for \((\bar{u}'\bar{v}')\) we can estimate \(b_{13}^{SD} \approx 1/1.4b_{13}^{3D}\), where \(b_{13}^{3D} = \langle u'[u_3]^2 \rangle / \langle u_3^2 \rangle\), and is usually referred to as the Townsend (1976) structure parameter. SV94 obtain a value of 0.13 within 0.2 < \(y/\delta < 0.4\) in a high Reynolds number turbulent boundary layer, decreasing to about 0.1 close to the wall and in the outer edge of the boundary layer. In the present data, the values of \(b_{13}^{SD}\) for runs 101 and 105 low elevations are 0.12, and they fall close to the expected range.

Before closing this section, we compare our Reynolds shear stresses with bottom stress estimates based on the profile method, using \(U(z) = u_l^k \ln(z) \) where \(u_l^k\) is the friction velocity and \(k = 0.41\) is the von Kármán constant. Individual velocity distributions have logarithmic behavior (Fig. 4b) as determined by the “goodness of fit” values \(r^2\) (Grant and Madsen 1986), which vary from 0.989 to 0.999. Table 3 shows \(u_l^k\) and the ratio \(u_l^k/((\bar{u}'\bar{w}'))_{map}^{1/2}\). The values of this ratio are close to 2 for all datasets, except for run 105, where \(u_l^k/((\bar{u}'\bar{w}'))_{map}^{1/2}\) is about 1.2. The discrepancy between these two estimates of friction velocity is not surprising since Fig. 6a provides evidence that the shear stress decreases with increasing elevation, and the fact that the measurements are performed in the outer layer. Thus, the present data highlight the shortcomings of the profile method in estimating bottom stress, and emphasize the level of subjectivity that one needs to exercise in determining whether the profiles are logarithmic or not. However, this ratio somewhat improves if we compare \(u_l^k\) with a value that one would obtain by extrapolating the measured shear stresses of runs 101 and 105 to the bottom using linear least squares fits. The extrapolated ratio becomes 1.2 for run 101 and 0.9 for run 105.

c. Dissipation estimates and spectral analysis

One-dimensional spatial energy spectra \(E_{11}\) and \(E_{33}\) of the instantaneous velocities are calculated in both the streamwise and wall-normal directions (\(k_1\) and \(k_3\) direction, respectively) using all the 3600 velocity fields in each dataset. The procedures include removal of a linear trend from the data, zero padding to extend each row or column to 64 points, and fast Fourier transform. The spectrum is calculated for each of the central seven rows or columns, and then averaged over them and over all the realizations in a dataset. In total, each plotted curve is an average of up to 25 200 spectra. A parametric study to evaluate the quality and reliability of

<table>
<thead>
<tr>
<th>Run No.</th>
<th>(u_l^k) (cm s(^{-1}))</th>
<th>(e_{map}^{SD} (s^{-1} \times 10^{-2}))</th>
<th>(\delta_{map}^{SD} (m))</th>
<th>(k_3^{SD} (m^2 s^{-2} \times 10^{-7}))</th>
<th>(\delta_{map}^{3D} (m))</th>
<th>(k_3^{3D} (m^2 s^{-2} \times 10^{-7}))</th>
<th>(\delta_{map}^{3D} (m))</th>
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* Data for runs 57 and 64 are taken from Nimmo Smith et al. (2005), mean elevation for these two runs are 0.55 and 2.45 m for run 57 and 64, respectively. ** Based on fit to energy spectrum.
the spectra obtained from the oceanic PIV data is presented in appendix A. In this paper, in addition to the SABSOON data discussed so far, we also present two sample spectra of data recorded during a previous deployment near the Large Ecosystem Observatory (LEO 15) site located off the New Jersey coast. The optical setup and averaging procedures of these spectra are identical to those of the present datasets. The currents at the LEO 15 site have been moderate to low, in comparison with the SABSOON conditions, and have also been exposed to oceanic swell. In the analysis that follows, we present two LEO-15 datasets obtained at mean elevations of 0.55 and 2.5 m above the bottom (runs 57 and 64, respectively). Some of the characteristics of the LEO 15 data are provided in Table 3, and detailed information can be found in NS05 (their runs D and E). Run 57 represents a tidal current of about 8 cm s⁻¹, which is typical of conditions near LEO 15, and run 64 represents very low mean flow conditions during the slack times of a tidal cycle (NS05). Figure 7 shows the distribution of transverse spatial spectra $E_{33}(k_1)$ in dimensional units. Only one of the SABSOON spectra (run 105) exhibits a slope that is close to $-5/3$. The LEO 15 spectra are clearly different.

1) DISSIPATION ESTIMATES

To present the measured spectra in a normalized form and compare them with the universal spectrum [see Eq. (1)], we need to calculate the dissipation rate and Kolmogorov length scale. Unfortunately, with the present PIV setup, we do not have sufficient resolution to resolve the smallest scales of the turbulence, which are needed for estimating the dissipation rate directly from spatial velocity derivatives (see discussion in Doron et al. 2001; NS05). Since the present resolution extends into the dissipation range (evidence will be provided shortly), direct calculations underestimate the dissipation, but should be within an order of magnitude of the correct values. Following Doron et al. (2001)—that is, assuming that the statistics of missing out-of-plane derivatives are the same as the in-plane values, but with slight modifications that would achieve the correct isotropic limit—an estimate of the direct ensemble averaged dissipation, $\epsilon_D$, is

$$
\epsilon_D(x, z) = 4\nu \left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \frac{3}{4} \left(\frac{\partial u}{\partial z}\right)^2 + \frac{3}{4} \left(\frac{\partial w}{\partial x}\right)^2 + \frac{3}{2} \left(\frac{\partial u}{\partial x}\frac{\partial w}{\partial z}\right)\right).
$$

The derivative calculations are performed using center differencing of instantaneous velocity at each point in a vector map (59 × 59 data points). Values of $\epsilon_{D_{\text{map}}}$ are presented in Table 3. Equation (5) is the only place where we use 2D PIV data to infer a quantity that requires 3D data. However, because of the resolution limit of present datasets, $\epsilon_D$ is not used in the subsequent analysis, as described below. Also, it should be noted that contrary to the direct estimates of Reynolds stresses, contamination by surface waves does not adversely affect the direct estimate of dissipation since the spatial scales of waves are $10^{-6}-10^{-5}$ orders of magnitude larger than those relevant to dissipation.

To evaluate how much $\epsilon_{D_{\text{map}}}$ is undervalued, we also estimate the dissipation rate using the G84 universal dissipation spectrum to account for the unresolved part of the dissipation range. This analysis provides $\epsilon_{IS}$, the spectra-based, “isotropic” estimate of dissipation rate. To perform this comparison we have scanned the original, full-sized plots used for generating the universal spectra figures in G84, as discussed in appendix B. Estimates of $\epsilon_{IS}$ are obtained by fitting the part of the dissipation spectrum that we can resolve, in the form of $(k_1 \eta_t)^3 E_{33}(k_1)/\left(\nu^3 \epsilon_{IS}\right)^{1/4}$ versus $\log(k_1 \eta_t)$, where $\eta_t = (\nu^3 \epsilon_{IS})^{1/4}$, to the G84 universal transverse dissipation spectrum with $\nu = 0.95 \times 10^{-6}$ m² s⁻¹, valid for the temperature range during the SABSOON measurements. Longitudinal and transverse spectra differ in isotropic turbulence, and are related to each other through relation (B2) given in appendix B. These differences are accounted for in the different forms of the universal
spectra. To obtain vertical distribution of $\varepsilon_{IS}(z)$, we calculate $E_{33}(k_1)$ for seven adjacent horizontal layers (seven rows each) and repeat the fitting process. Results over the central seven rows are presented in Fig. 8. We choose to perform spectral fits and present the results using the log-linear, variance-preserving plot of the dissipation spectrum because of its high sensitivity in determining the degree to which data fit the universal curve. This approach emphasizes the high wavenumber range, which dominates the dissipation rate.

Henceforth, we take the sample area average, $\varepsilon_{ISmap}$, as our best estimate of dissipation rate, and their values are given in Table 3. For all the SABSOON data, $\varepsilon_{ISmap}$ falls between 40% and 60% of the $\varepsilon_{ISmap}$. The LEO 15 data do not fit the universal dissipation spectrum because the turbulence levels there are too low to have an inertial range (NS05). The Kolmogorov length based on $\varepsilon_{ISmap}$ that is, $\eta_{IS} = (\nu^3/\varepsilon_{ISmap})^{1/4}$; the Taylor microscale $\lambda^*$ defined as $\lambda^* = 15\nu (u'v')_{map}/\varepsilon_{ISmap}$; and the Taylor microscale Reynolds number, $Re_\lambda = (u'v')_{map}^{1/2} \lambda^* \nu$, are also presented in Table 3. This Reynolds number is commonly used in analysis of turbulent flows (Pope 2000), and may be interpreted as the ratio of the large-eddy time scale (mean shear time scale) to the time scale of strain rate fluctuations (Tennekes and Lumley 1972). Note that peak in dissipation spectrum given in Fig. 8 occurs at $k_1 \eta_I \approx 0.25$, a length scale of about 25$\eta_I$, which is about equal to the calculated values of the Taylor microscale. The values of $Re_\lambda$ at LEO 15 are less than 100, no matter what kind of method is used for estimating the dissipation rate; that is, they are well below the levels that could have universal properties (Pope 2000). For the SABSOON data, $Re_\lambda$ ranges between 240 and 665. We have also calculated $\varepsilon_{ISmap}$ by fitting the normalized dissipation spectrum of $E_{11}(k_1)$ to the G84 transverse spectrum. Because of the anisotropy that will be discussed below, the $E_{11}(k_1)$ based dissipation estimates are about 40% larger than those obtained from $E_{33}(k_1)$.

2) Normalized energy spectra

Figure 9 shows the distributions of $E_{33}(k_1)$, $E_{33}(k_3)$, $E_{11}(k_1)$, and $E_{11}(k_3)$, respectively, all normalized using $\varepsilon_{ISmap}$ and $\eta_{IS}$ along with the G84 spectrum. Since we cannot obtain $\varepsilon_{IS}$ for the LEO 15 data using the same process, we estimate it by fitting their $E_{33}(k_1)$ (and using $\nu = 1.23 \times 10^{-6}$ m$^2$ s$^{-1}$) with the G84 energy universal spectrum at high wavenumbers and provide them in Table 3. Collapse of $E_2(k_1)$ (Fig. 9a) onto the universal curve in the dissipation range is a consequence of our fitting. For $E_{33}(k_1)$ and $E_{11}(k_3)$ at low wavenumbers the energy increases and tends toward the universal spectrum with increasing $Re_\lambda$. At $Re_\lambda > 382$ the low wavenumber part of $E_3(k_1)$ already has a slope of about $-\frac{5}{3}$, while part of $E_1(k_3)$ has a slope slightly smaller than $-\frac{5}{3}$ at $Re_\lambda > 297$. Note that in laboratory-generated turbulence we obtain a clear $-\frac{5}{3}$ slope region at $Re_\lambda > 150$ using the same analysis procedure (Liu et al. 1994; NS05).

In the longitudinal spectra, $E_{55}(k_1)$ and $E_{11}(k_1)$, a range of wavenumbers resembling an inertial range is present only for the largest $Re_\lambda$ (run 105). At low wavenumbers they both increase with increasing Reynolds number. At the transition between inertial and dissipation range, both longitudinal spectra develop a “bump” starting from $k_1 \eta_{IS} \approx 0.07$ and $k_3 \eta_{IS} \approx 0.08$ for $E_{11}(k_1)$ and $E_{55}(k_3)$, respectively. Existence of spectral bumps at the transition range has been seen before in several studies, for example, Champagne et al. (1977) in atmospheric turbulence, SV94 in wind tunnel data, oceanic flows by Doron et al. (2001), and NS05. Sreenivasan (1995) provides a comprehensive list of wall-bounded flows in which measured spectra display spectral bumps. The bumps in oceanic flows are more pronounced than those reported in other environments, and are particularly evident in $E_{11}(k_1)$, and smaller but still substantial in $E_{33}(k_3)$. They are not caused by our analysis procedures, as confirmed by repeating them, from data acquisition through the entire spectral analysis, using laboratory PIV data of locally isotropic turbulence. There are no spectral bumps in the laboratory data (NS05).
3) CONDITIONAL SAMPLING

In an attempt to identify what affects the bumps, we performed an extensive series of conditional samplings based on several criteria involving large and small-scale features of the flow. The basis for “large scale” conditional sampling includes $\left[\frac{\tilde{u} \cdot \tilde{u}}{H^{1/2}}\right]_{\text{map}}$, $\left[\frac{\tilde{u} \cdot \tilde{z}}{H^{1/2}}\right]_{\text{map}}$, and $\left[\frac{\tilde{\omega} \cdot \tilde{u}}{H^{1/2}}\right]_{\text{map}}$, where the tilde indicates spatial filtering of the instantaneous velocity at a scale of 15 vector spacing, which for SABSOON data is 81 mm. Parameters representing small scales include the instantaneous direct dissipation rate averaged over sample area $e_{\text{map}}$ and the instantaneous RMS value of the high-pass, spatially filtered horizontal velocity at five vector spacings. The latter are particularly chosen since they match with the wavenumber at which peak of the spectral bumps occur. For each parameter the conditional samplings are based on two thresholds: a high threshold with values exceeding the mean + 1 standard deviation, and a low threshold with values below median. These thresholds are selected to include a sufficient number of cases in each set. Typically, the percentage of instantaneous vector maps satisfying the high and low thresholds are 11% and 50%, respectively, resulting in over 60% of the data being used in the analysis. To obtain normalized spectra for each subset, the conditional $e_{\text{ISmap}}$ and corresponding $\eta_{\text{IS}}$ are calculated in the same way as discussed before.

For each of the conditioned $E_{ij}(k_i)$ we have found that the specific parameters used for sampling, small or large scale based $\left[\frac{\tilde{u} \cdot \tilde{z}}{H^{1/2}}\right]_{\text{map}}$ or $e_{\text{map}}$ have minor impact on the distribution of normalized energy levels at all scales for the same threshold level (high or low). However, there are substantial differences in the distribution of normalized spectral energy between the high and low thresholds for the same sampling criterion. For example, in Fig. 10 we show the normalized $E_{33}(k_3)$ and $E_{11}(k_3)$ for runs 101, 103, and 105, conditioned on $\left[\frac{\tilde{\omega} \cdot \tilde{u}}{H^{1/2}}\right]$.
The Taylor microscale Reynolds numbers of subsamples are 263 (run 101), 310 (103), and 642 (105), for the high thresholds and 196 (101), 157 (103), and 537 (105) for the low thresholds. All spectra conditioned on high threshold reasonably follow a $\frac{5}{3}$ slope and they are significantly more energetic than the low threshold data at low wavenumbers. For the latter, substantial deviations from the universal curve occur for runs 101 and 103. At high wavenumbers the differences between high and low thresholds data diminish, and $E_{11}(k_1)$ forms a spectral bump at $k_1\eta_{IS} > 0.07$. Clearly, the spectral bumps exist regardless of the criteria or thresholds selected for conditional sampling. Contrary to the NS05 conclusion that bumps are less evident in spectra conditioned on high threshold, for the SABSOON data the bumps persist. Only for run 105, which has the highest Reynolds number, the spectral bump is reduced slightly at high threshold (Fig. 10b). The slope of $E_{11}(k_1)$ at $k_1\eta_{IS} > 0.07$ is approximately equal to $(k_1\eta_{IS})^{-1}$, as demonstrated in Figs. 9c and 10b. This result is consistent with the conclusion of She and Jackson (1992) who obtained a $k^{-1}$ slope for experimental data at moderate to high Reynolds numbers. They postulated that this trend indicates energy pileup near the peak dissipation wavenumber, consistent with the theory of Falkovich (1994).

An analytical turbulence closure model introduced by Yakhot and Zakharov (1993) leads to the prediction that the $k^{-\frac{5}{3}}$ inertial law has to be supplemented by a $k^{-1}$ term, leading to the appearance of a spectral bump. It is not clear to us as to what specific structure in the velocity field produces the bumps in $E_{11}(k_1)$ and to a lesser degree in $E_{33}(k_3)$, but not in the transverse spectra. The impact of the bump can be easily observed in the instantaneous velocity distributions. Figure 11 shows a sample distribution of instantaneous horizontal and vertical velocity components along a single row and column in the center of the sample area, after subtracting the line mean and detrending as used in the FFT calculations. Clearly, along the $x$ direction, the horizontal velocity fluctuations are much more energetic (jittery) at small scales in comparison with the vertical velocity fluctuations. Conversely, along the $z$ direction, the vertical velocity component is more energetic (jittery) than the horizontal component. The spatial extent of this jitter is about 1.5–2 cm, corresponding to three–four vector spacings. The large-scale variations of the two velocity components appear to be similar, consistent with the spectra. Typical peak to trough variation in magnitude of velocity jitter is approximately $5 \times 10^{-3}$ m s$^{-1}$, which corresponds to a displacement of 0.25 pixels. Although this level is close to our uncertainty level, we do not believe that the jitter is a result of error in measurements. If it was, it would appear in both velocity components in both directions. Furthermore, the very same equipment has been utilized in atmospheric measurements above corn canopy at $Re_a = 3000$ (van Hout et al. 2007), and show that the turbulence there is isotropic at small scales, and does not have any spectral bumps. The bumps in oceanic longitudinal spectra are real, but as yet we have no complete explanation for them. From a detailed analysis of the wave-phase dependence of the constituents of the subgrid-scale energy fluxes in the LEO 15 data, Nimmo
Smith et al. (2007) show that the combined action of the mean and wave-induced strain may suppress the cascading process in the horizontal component, and postulate that this process leads to enhancement of the bumps. However, to study the dynamics of flow structures at scales of less than 2 cm, we clearly need higher-resolution data, as we now have from recent ocean measurements and will report on them in the future.

4) Anisotropy

To examine the levels of anisotropy in the present data, Fig. 12 shows the ratios $E_{11}(k_1)/E_{33}(k_1)$ and $E_{11}(k_3)/E_{33}(k_3)$, which should be equal to unity at all scales where local isotropy is expected to hold. The values for the longitudinal spectra are below unity at low wavenumbers, but increase significantly with $k_{LS}$ as the spectral bumps develop. Values of $E_{11}(k_1)/E_{33}(k_3)$ for the SABSOON and LEO 15 data differ. At high wavenumbers, the SABSOON values seem to reach a plateau of 1.5–1.6, while for the LEO 15 data, the largest ratio is about 3. The values of $E_{11}(k_3)/E_{33}(k_1)$ are always above 1. They are high at low wavenumbers, decrease to a minimum of just above 1 to 1.4 in the range $0.02 < k_{LS} < 0.05$, and then increase.

Fig. 11. Sample instantaneous horizontal (triangles) and vertical (circles) velocity components along a single (a) row and (b) column in the center of the sample area. Data are from run 103.

Fig. 12. (a) Ratio of longitudinal spectra; (b) ratio of transverse spectra. For key to symbols, see Fig. 7.
again. The largest deviation from isotropy at low wavenumbers occurs close to the bottom, that is, runs 101 and 105, and is most likely associated with the mean shear (see below). At high wavenumbers, the ratios of transverse spectra are consistent with those of the normal spectra, even without bumps. Extending the conclusions based on the 2D anisotropy tensor (Figs. 6b), the spectral analysis indicates that turbulence anisotropy exists at all scales. SV94 show that to achieve full local isotropy in boundary layers at wavenumbers within the inertial range and at Re, > 1000, the ratio of the Kolmogorov to mean shear time scales S(ν/κ)1/2 should be less than 0.01. Here S is mean strain rate magnitude defined as S = (2SxSy)1/2. Note that S(ν/κ)1/2 is proportional to (Re, )−1 if turbulent kinetic energy (TKE) production is approximately equal to dissipation (Tennekes and Lumley 1972; SV94). Evaluating S from the available in-plane quantities, S(2D) = [(2Sx + 4Sy)2 + 2(3Sx)2]1/2 (map, with Sx = 1/3(∂U/∂x + ∂U/∂y), and Ss = (1/2)(∂(w′)2)/∂z) we obtain values of S(2D)/(ν/ε(2D)) in the range between 0.01 to 0.03. Values of S(2D) are provided in Table 3. Furthermore, Durbin and Speziale (1991) argue from theoretical grounds that local isotropy is inconsistent with the presence of mean shear in high Reynolds number turbulent boundary layer, and consequently the necessary condition for local isotropy is Sk/ε ≪ 1, where k is the turbulent kinetic energy. This parameter represents the ratio of time scale of large-scale turbulence, k/ε, to the time scale of mean deformation 1/S. We calculate S(2D)k(2D)/ε(2D), where k(2D) is the kinetic energy evaluated from the in-plane normal stresses, k(2D) = 1/2⟨(u′u′) + (w′w′)⟩ (map). Including the approximate expression for the normal out-of-plane stress, ⟨v′v′⟩, it follows that k(2D) = 1.4k(2D). As shown in Table 3, S(2D)k(2D)/ε(2D) varies between 1.6 and 2.8 near the bottom to 0.6 at higher elevations. Thus, one should expect a reduction in large-scale anisotropy with increasing elevation consistent with the trends shown in Figs. 6 and 12. If k(3D) is included in calculations, this parameter becomes even larger, for example, S(2D)k(3D)/ε(2D) = 3.9 for run 105, further implying that the present anisotropy, at least at large scales, is a consequence of the mean shear.

Another possible source of anisotropy is stratification of the water column. The G84 measurements in a stably stratified flow show that as the turbulence decays, anisotropy due to stratification appears first at the low wavenumber end of spectra. In the present data at the higher elevations, turbulence is decaying as discussed in the next section. Yet we do not see an increase in the level of large-scale anisotropy with decreasing Re. In fact, at low Re, the large scales are the least anisotropic, even when the spectra deviate form the universal slope (Fig. 12). Thus, we do not suspect that stratification plays a major role in the present data. To confirm this assertion we estimate the buoyancy frequency, N = (−gρb−1∂(ρ)/∂z)1/2, following the G84 approach. Although we do not have continuous local values for d(T)/dz (here T is temperature) over the sample area, we can estimate it using our system-mounted CTD data just before and after changing elevations, along with the depth records. The temperature and salinity data of the ship’s CTD casts (Fig. 3) are used to determine the slope of temperature–salinity curves. Resulting estimates for N are 1.62 × 10−2 and 1.74 × 10−2 rad s−1 (10 cycles h−1), and the corresponding buoyancy wavenumbers (inverse Osmidov scale) kO = (N3/ε(2D))1/2 are 0.617 and 0.737 rad m−1 for runs 102 and 106, respectively. G84 formulate an anisotropy parameter I = 1/(k0/ε(2D)), as the ratio of Osmidov to the Kolmogorov length scale to classify departure from isotropy. The present values of I are 2489 and 3080, and they fall in a range where the G84 spectra exhibit inertial subranges and satisfy the conditions for local isotropy. Thus, the anisotropy present in our data does not seem to be a consequence of stratification. Also, buoyancy-related anisotropy in G84 does not extend to small scales, in contrast to the present results, where anisotropy in bottom boundary layer is particularly high at dissipative scales.

Last, we examine the effect of Re on anisotropy at small scales by calculating second-order statistics of the measured local velocity gradients at the smallest resolved scales (10–13 Kolmogorov lengths). Local isotropy (Pope 2000) implies that

$$\left\langle \left( \frac{\partial u}{\partial x} \right)^2 \right\rangle_{\text{map}} = \frac{1}{2} \left\langle \left( \frac{\partial w}{\partial z} \right)^2 \right\rangle = \frac{1}{2} \left\langle \left( \frac{\partial u}{\partial z} \right)^2 \right\rangle = \frac{1}{2} \left\langle \left( \frac{\partial w}{\partial x} \right)^2 \right\rangle.$$ (6)

As illustrated in Fig. 13 the ratios ⟨(∂u/∂z)2⟩map/⟨(∂w/∂z)2⟩map and ⟨(∂u/∂z)2⟩map/⟨(∂w/∂x)2⟩map are higher than unity but decrease with increasing Reynolds number. Each point in the plot is an average over 2.5 × 106 values for the SABSOON data, and 1.5 × 106 values for the LEO 15 data. Clearly, the results indicate a tendency toward small-scale isotropy with increasing Re. Thus, it is plausible that levels of anisotropy in the present datasets are related to the low values of attained Re. However, measurements in a wind tunnel by Garg and Warhaft (1998) show that for homogeneous shear flow and Reynolds number in the range 156 < Re < 390 anisotropy is persistent both at inertial and dissipation scales. Also, SV94 show that local isotropy in turbulent boundary layer is achieved at Re > 1000.
d. Production dissipation balance

The in-plane contribution to the turbulent kinetic energy production rate $P^{2D}$ is

$$P^{2D}(z) = -\langle u'u' \rangle \left( \frac{\partial U}{\partial x} - \frac{\partial W}{\partial z} \right) - \langle w'w' \rangle \left( \frac{\partial W}{\partial z} \right).$$

(7)

Since for most of the present data, the plane of measurement is aligned with the mean flow, Eq. (7) should also provide a reasonable estimate for the total production rate $P = -\langle u'u' \rangle S_0$. The profiles of $P^{2D}(z)$ and $e_{15}(z)$ are plotted together in Fig. 14. Each point is an average of seven rows. We also include error bars on the appropriate $P^{2D}$ estimates due to the following sources: convergence of structure functions, misalignment of PIV plane, and contributions to the production rate due to the only out-of-plane component $-\langle u'u' \rangle (\partial V/\partial y)$, that we can estimate. As discussed before $\langle u'u' \rangle$, $\langle w'w' \rangle$, and $-\langle u'w' \rangle$ are underestimated by 10%, 15%, and 35% for run 103; by 5%, 0%, and 10% for run 106; and by 15%, 15%, and 45% for run 107, respectively. The real $P^{2D}$ for these runs are 34%, 6%, and 28% higher. This effect diminishes at elevations lower than 1 m since the stress estimates there are converged, and the alignment of PIV plane with the mean flow is better. If we exclude the out-of-plane component, $-\langle u'u' \rangle (\partial V/\partial y)$ in estimating $P$, with $-\langle u'u' \rangle$ approximated as $\langle u'u' \rangle = 0.4(\langle u'u' \rangle + \langle w'w' \rangle)$ and $(\partial V/\partial y) = -\langle u'u' \rangle (\partial W/\partial z)$ the production rate for run 105 increases by 13%, and for run 101 it decreases by 8%. For all other measurements the decrease in production rate by including $-\langle u'u' \rangle (\partial V/\partial y)$ is significant: 38% and 73% for runs 102 and 103, and 42% and 57% for runs 106 and 107, respectively. We do not account for the other missing out-of-plane contributions, since we do not have data to estimate them.

The $P^{2D}(z)$ profiles of runs 101 and 105 differ in magnitude and both exhibit a substantial variation with elevation. For the other measurements, the values of $P^{2D}(z)$ are of the comparable magnitude, and display a slow decrease with height. The dissipation rate profiles, $e_{15}(z)$, also decrease with elevation but at the slower rate, and remain significant even far from the bottom. Sample area averages, $(P^{2D}/e_{15})_{map}$, are given in Table 3. At all measurement heights, the dissipation exceeds production and $(P^{2D}/e_{15})_{map}$ ranges from 0.1 at high elevations to 0.5 near the bottom. The usually assumed balance of $P^{2D} \approx 1$ appears to be correct only for the lowest height of run 105, which is recorded at the beginning of the tidal deceleration phase, where at $z = 32$ cm $P^{2D}(z)/e_{15}(z) = 0.7$. At the same $z$, for run 101 measured at the beginning of the acceleration phase, $P^{2D}(z)/e_{15}(z) = 0.5$. This trend is expected, since the same $z$ for these two runs does not correspond to the same appropriately scaled location in the boundary layer. If, as Fig. 6a shows, the stresses scale with $U_{ADCP}^2$ and $\partial U/\partial z$ scale with $U_{ADCP}^2/\delta$, then $P(z) \sim U_{ADCP}^3/\delta$ (Bradshaw 1967). Unfortunately, we do not have $\delta$. However, this scaling implies that $\delta_{101}/\delta_{105} = (P^{2D}_{101}/P^{2D}_{105})^{1/3}$. 

Fig. 14. The distribution of energy dissipation $e_{15}(z)$ (open symbols) and TKE production $P^{2D}(z)$ rates (full symbols). For key to symbols see Fig. 6. Contributors to error bars are described in section 3d.
$P_{105}^{2D}/\varepsilon_{105}$ map $U_{ADCP~105}^2/U_{ADCP~101}^2$ is about 1.8. Thus, run 105 represents conditions to correspond to a lower dimensionless elevation than those of run 101. Based on Bradshaw (1967) laboratory data, the production rate decreases rapidly with elevation in the outer parts of a boundary layer (see also discussion below).

To elucidate further the trends in our data at the lowest elevation, in Figs. 15a and 15b we present the contributions to $P^{2D}(z)$ from individual terms in Eq. (7) together with corresponding dissipation, $\varepsilon(z)$. As expected, $-(u'w')(\partial U/\partial z + \partial W/\partial x)$ is the dominant contributor to the production rate, but contributions from the normal components are not insignificant. Contribution from $-(u'w')(\partial W/\partial z)$ is positive, primarily because of negative $\partial W/\partial z$ (see Table 2), while $-(u'u')(\partial U/\partial z)$ persistently has negative contributions. The third (approximated) out-of-plane normal component, $-(u'u')(\partial V/\partial y)$, is small at this elevation. If for comparison, we estimate $P_{\text{heat}} = -(u'w')(\partial U/\partial z)$, as is customary in physical oceanography literature (e.g., Thorpe 2005; Sanford and Lien 1999), it results in sample area averages ($P_{\text{heat}/\text{map}}$) of 0.8 and 0.5 for runs 105 and 101, respectively. Thus not accounting for the contribution from the normal components in Eq. (7) and neglecting $\partial W/\partial x$ results in an increase of the production–dissipation ratio by 60% for run 105 and by 25% for run 101. At higher elevations (not shown), all terms in (7), including the out-of-plane component, become equally significant as $(\partial U/\partial z)$ and the shear stresses diminish. However, $P_{\text{heat}}$ has no significant impact on the production dissipation ratios presented in Fig. 14.

It is difficult to compare the profiles in Figs. 14 and 15 with the published laboratory data (e.g., Bradshaw 1967) or numerical simulations (e.g., Spalart 1988; Kim et al. 1987) because of substantial variations in trends with Reynolds number. In a boundary layer above a smooth wall, $P > \varepsilon$ below 50 wall units. At higher elevations the reported trends differ. In Kim et al. (1987) there is a region with $P = \varepsilon$, while in Spalart (1988) and in Bradshaw (1967) $P$ is slightly larger than the dissipation up to $z/\delta \approx 0.8$. At higher elevations $P < \varepsilon$ in Spalart data, and $P > \varepsilon$ in Bradshaw results, but they both reduce to negligible levels at the edge of the boundary layer. Using data provided by SV94 for $y/\delta = 0.36$ ($R_\alpha = 1450$) and 0.46 ($R_\alpha = 600$), one can calculate the production rate and compare them with $\varepsilon$. The results are $P_{105} = 0.8$ and 0.9, respectively. Thus, it appears that our high Reynolds number, near-bottom result (i.e., run 105) is consistent with the SV94 data, and further reinforces the fact that measurements are performed in the outer portions of the boundary layer. At higher elevations, existence of $(P_{105}/\varepsilon_{105})_{\text{map}}$ imbalance has been measured in several field experimental studies, for example, Sanford and Lien (1999) in a tidal channel, and Shaw et al. (2001) on the New England shelf. Both studies show that dissipation exceeds production in the outer parts of the boundary layer.

We cannot provide substantiated explanation for the large discrepancy between production and dissipation at high elevations. We believe that $P > \varepsilon$ near the
bottom because of interaction of currents and waves with the rough bottom topography. However, our measurements are performed well above the wave boundary layer, and also above the inner part of the mean current boundary layer, as the Reynolds stress profiles indicate. Advection of energy by the mean flow or by waves, as well as turbulent and pressure transport must account for the discrepancy between the production and dissipation rates. Also, wave-induced strains may contribute to the turbulent kinetic energy production rate, as discussed in Reynolds and Hussain (1972). We have already shown that buoyancy does not play a significant role in the present measurements.

4. Summary and conclusions

In this paper we present results of PIV measurements conducted over a half tidal cycle, covering 2.5 m of the bottom boundary layer on the continental shelf, in a water depth of 23 m. The measurements took place in the vicinity of SABSOON located along the South Carolina and Georgia coast. Flow conditions included tidal currents with speeds from 10 to 50 cm s\(^{-1}\), surface waves with typical period of 8 s and wave-induced horizontal velocity amplitudes of 4-5 cm s\(^{-1}\). Although the water column was stably stratified well above the bottom, calculations of the buoyancy frequency and Osmidov scale show that stratification does not play a major role in the near-bottom dynamics.

Our objective has been to obtain distributions of mean flow, Reynolds stresses, and energy spectra, as well as production and dissipation rates. We use structure functions to calculate the Reynolds stresses, taking advantage of spatial distribution provided by the PIV data. The range of Taylor microscale Reynolds numbers for the SABSOON measurements is 240–665, but we also use two datasets from a previous deployment with lower Reynolds numbers. We include results from measurements at the LEO 15 site to show variability in spectral trends, anisotropy levels, and Reynolds numbers that we believe are characteristic of the continental shelf, bottom boundary layer flows. Runs 57 and 64 reflect typical calm weather conditions in coastal waters. Near-bottom currents during the SABSOON deployment (especially during the peak tidal flow) are the strongest currents our group has encountered, and in particular, run 105 \(Re_e = 665\) is the largest Reynolds number we have obtained to date in our oceanic measurements. We would need to perform measurements in tidal channels, or away from boundaries, in order to obtain higher Reynolds numbers. For example, G84 measurements performed in a tidal inlet and away from the boundaries have \(Re_e\) of about 2000 (Gargett 1985).

The vertical gradients of the mean streamwise velocity decrease with increasing elevation, but do not vanish at the highest measurement elevation of almost 3 m. The mean current profiles, measured by ADCP, over the entire water column exhibit a tide-phase-dependent mean shear. As a result there is no clear demarcation between the boundary layer edge and the sheared flow above it, and we do not have a method to determine the bottom boundary layer thickness. All the mean velocity profiles have logarithmic behavior despite the fact that shear stress decreases with increasing elevation, and that except for present lowest elevation, the production rate is significantly lower than the dissipation rate. Consequently, for most cases the estimates of \(u'\), obtained from the profile method, are larger in comparison with the estimates of friction velocity using the measured shear stress. Only for the run 105 the two estimates are comparable, where \(u' = (-u'w')_{map}^{1/2} = 1.2\). This result is consistent with the fact that only for this run are the three fundamental properties of log–law region approximately met (Pope 2000): exhibit logarithmic profile, production approximately balances dissipation rate, and \(-\langle u' w' \rangle/k \approx 0.3\) (from Fig. 6b; 2\(b_{13}^{\text{ID}} = -\langle u' w' \rangle/k^{\text{ID}} = 0.24\)). It is also worth noting that the mean streamwise velocity in the bottom boundary layer is not logarithmic for the flow conditions characterized by mean currents of the same magnitude or weaker as the wave-induced motions, as experienced at LEO 15 site (Nimmo Smith et al. 2002).

Quantitative comparisons of normalized stresses and production–dissipation balance with laboratory results show agreement in magnitude and trends, suggesting that our measurements are performed within 0.4 < \(z/\delta \times 0.7\). Furthermore, the fact that the turbulent stresses scale reasonably well with \(U_{\text{ADCP}}^2\) supports the notion that our measurements are performed in the outer portion of the boundary layer. Within our range of measurements, the vertical structure of the shear stress profile is such that shear stress decreases with elevation, and if a constant stress layer exists, it is confined to within the bottom 1% (25 cm) of the water column for which we do not have data. Since the vector spacing in our measurements is 10–13 Kolmogorov lengths, we use the Gargett et al. (1984) universal dissipation spectrum to account for the unresolved part of the dissipation range, and from it we estimate \(e_{\text{is}}\). Thus, \(e_{\text{is}}\) is based on an assumption of isotropy of dissipative scales. However, we show, using several means, that the turbulence is anisotropic at all measured scales, especially in the resolved part of the dissipation range. Other methods to examine this issue, such as the distribution of the correlation coefficient spectrum \(R_{13}\) (not shown here; SV94; Antonia and Raupach 1993), lead to the same result. This conclusion raises a ques-
tion on use of $e_{1s}$ as our estimate of dissipation rate. To answer this, note that our instantaneous velocity distributions cover about 40%–60% of the dissipation spectrum since our spatial resolution is 10–13 Kolmogorov lengths (Table 3). Thus, we need to find means to extend the available information in order to provide the “best estimate” of dissipation rate. The only physically meaningful method that we can think of is to fit the measured data onto the universal dissipation spectrum. The values of $e_{1s}$ are typically twice those based on the underresolved direct estimates, consistent with the fraction of the dissipation spectrum that we cover. We choose to fit $E_{11}(k_1)$ since it provides the lowest estimates. Estimates based on $E_{11}(k_1)$ are 40% higher. Thus, the present values of $e_{1s}$ are lower bounds, but they are probably within a factor of 2 from the correct values. These results encourage us to increase our resolution in oceanic PIV measurements. Indeed, in a subsequent field deployment we have increased the magnification of one sample area to a level that resolves the Kolmogorov scale.

At large scales, the anisotropy increases as the bottom is approached. We believe the cause of this anisotropy is the mean shear due to wall presence, which occurs in all boundary layers as shown by comparing the time scale of mean shear and turbulence $S^2Dk^2D/\epsilon_{1s}$map. Still, with increasing Reynolds numbers, the spectral slopes at low wavenumbers tend toward $-\gamma$, and to the G84 universal spectrum. At small scales $E_{11}$ is consistently more energetic than $E_{33}$ whether we compare longitudinal or transverse spectra. Their ratio decreases with increasing Reynolds number, but are clearly above one over our entire range of measurements. Furthermore, the longitudinal spectra have bumps, with slope of approximately $-1$ at the transition between inertial to dissipation scales. Conditional samplings based on several large- or small-scale parameters demonstrate that these bumps persist. The physical manifestation of bumps is evident in velocity distributions as a jitter with scales of 1.5–2 cm. However, being only three–four vector spacings wide, we cannot resolve the detailed flow features at these scales. Higher-resolution data obtained in subsequent field measurements will be used for revisiting this issue in the future.

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### APPENDIX A

### Limitations of Spectral Analysis Based on PIV Data

The spatial resolution of PIV is limited by the magnification and the resolution of the CCD array. The resolved range of scales is determined by the size of the sample area for the largest scale, and by the size of the interrogation window for the smallest scale, although the vector spacing can be smaller by using overlapping interrogation windows. For oceanic flows that rely on natural seeding, the size of the interrogation window is determined by the particle concentration that provides reasonable signal-to-noise ratio for the correlation analysis (Raffel et al. 1998).

The velocity calculated for each interrogation window is based on the average displacement of all the particles within the window (e.g., $64 \times 64$ pixels); that is, it can be thought of as a low-pass-filtered version of the true local fluid velocity (Westerweel 1997). A recent study by Foucaut et al. (2004) investigates the noise in spectra generated by PIV data. They use results of PIV measurements in a turbulent boundary layer to calculate energy spectra and compare them with hot-wire results. They confirm that the PIV interrogation window behaves as a low-pass filter, and conclude that the noise is white. The seeding density and distribution are identified as major contributors to the noise.

We have performed a parametric study to investigate effects of the interrogation window size and overlap between windows on the quality of energy spectra obtained in the oceanic PIV measurements. The test parameters are identified in Table A1, and we use run 105 data. Figure A1 shows $E_{11}(k_1)$ in dimensional units for the six cases and the spectra is calculated as discussed in section 3.

<table>
<thead>
<tr>
<th>No.</th>
<th>Interrogation window (pixels × pixels)</th>
<th>Vector spacing (pixels)</th>
<th>Percent overlap</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$128 \times 128$</td>
<td>64</td>
<td>50</td>
</tr>
<tr>
<td>2</td>
<td>$128 \times 128$</td>
<td>32</td>
<td>75</td>
</tr>
<tr>
<td>3</td>
<td>$64 \times 64$</td>
<td>64</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>$64 \times 64$</td>
<td>32</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>$64 \times 64$</td>
<td>16</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>$32 \times 32$</td>
<td>32</td>
<td>0</td>
</tr>
</tbody>
</table>

All spectra overlap at large scales but the energy at intermediate and small scales decreases with increasing window size. The deviations occur for two reasons: low-pass filtering, and decrease in noise with increasing...
number of particles in the window. The 32 × 32 interrogation window, which we regularly use in the laboratory, is clearly too noisy with the oceanic seeding. On the other hand, the 128 × 128 window, low-pass filter removes too much energy at intermediate scales, explaining why we have selected 64 × 64 window as an optimal interrogation window size.

As the level of overlap between interrogation window increases to 75%, a second lobe appears at small scales, which is a characteristic of spatial box filtering. Thus, the overlap between windows exceeding 50% does not increase the spatial resolution of the measurements. This conclusion is consistent with Foucaut et al. (2004). On the other hand, when 50% overlap is used, the results have less high wavenumber noise in comparison with analysis with no overlap. Thus, 50% overlap seems to be an optimal choice.

APPENDIX B

Digitization of the Gargett et al. (1984) Spectra

To use the Gargett et al. (1984) universal spectra as the basis for comparison with the experimental data, we have scanned and digitized the original full-sized plots (Figs. 11a and 11b) from that paper. The following eighth-order polynomial, (Figs. 11a and 11b) from that paper. The following eighth-order polynomial,

\[
E_{11}(k_1,\eta) = \sum_{n=0}^{8} B_n \left[ \log_{10}(k_1\eta) \right]^n, \quad (B1)
\]

is then least squares fitted to the digitized plots, with the coefficients given in Table B1. These results are valid in the range \(-2.7 \leq \log_{10}(k_1\eta) \leq 0.5\).

An alternative way to obtain transverse spectra, \(E_{22}(k_1\eta)\) and \(E_{33}(k_1\eta)\), is to use the isotropic relation (Pope 2000) and the least squares results for \(E_{11}(k_1\eta)\):

\[
E_{22}(k_1\eta) = E_{33}(k_1\eta)
\]

\[
= \frac{1}{2} \left[ E_{11}(k_1\eta) - (k_1\eta) \frac{\partial E_{11}(k_1\eta)}{\partial (k_1\eta)} \right]. \quad (B2)
\]

To verify the results, note that because of local isotropy the universal dissipation spectrum satisfies

\[
\int_{0}^{\infty} (k_1\eta)^2 E_{11}(k_1\eta) d(k_1\eta) = \frac{1}{15} \quad \text{and}
\]

\[
\int_{0}^{\infty} (k_1\eta)^2 E_{33}(k_1\eta) d(k_1\eta) = \frac{2}{15}, \quad (B3)
\]

which can be obtained from \(\varepsilon = 15\nu(\partial u/\partial x)^2 = 15/2\nu(\partial u/\partial x)^2\) (Champagne 1978). Integrating our least squares results within the limits of validity of coefficients, we obtain 0.0674 and 0.1351 for the longitudinal and transverse ratios. If the transverse spectrum is calculated from Eq. (B2), the ratio is 0.1349. Thus, the digitized spectra give values that are only 1% larger than the theoretical isotropic value. In this paper we use least squares results for \(E_{11}(k_1\eta)\), and the transverse spectrum is calculated using Eq. (B2).

REFERENCES


