Turbulence Characteristics and Dissipation Estimates in the Coastal Ocean Bottom Boundary Layer from PIV Data

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Abstract

Turbulence characteristics in the coastal ocean bottom boundary layer are measured using a submersible Particle Image Velocimetry (PIV) system with a sample area of 20 x 20 cm. Measurements are performed in the New York Bight at elevations ranging from about 10 cm to about 1.4 m above the sea floor. Data for each elevation consists of 2 minutes of image pairs recorded at 1 Hz. The data provides instantaneous spatial velocity distributions within the sample area, and is extended to larger scales using Taylor’s hypothesis.

The vertical distribution of mean velocity indicates the presence of large-scale shear even at the highest measurement station. It also undergoes variations with time scales longer than the present data series.

The turbulence spectra calculated from the data cover about three decades in wavenumber space, and extend well into the dissipation range. The results indicate, that the turbulence near the bottom is anisotropic, not only at the scale of the energy containing eddies, but also in the inertial and dissipation ranges. The vertical component of velocity fluctuations at energy containing scales is significantly damped as the bottom is approached, while the horizontal component maintains a similar energy level at all elevations.

Different methods of estimating the turbulent energy dissipation are compared. Several of these methods are possible only with 2-D data, such as that provided by PIV, including a “direct” method, which is based on measured components of the dissipation tensor. Estimates based on assumptions of isotropy exhibit deviations from those based on the “direct” method.
1. Introduction

In the coastal ocean the momentum and energy balances are influenced by several parameters, among which the bottom shear stress and the dissipation rate are of particular significance. The bottom shear stress affects the circulation directly and also generates turbulence that diffuses into the flow. The dissipation rate is a controlling mechanism for the entire turbulent energy budget. Thus, understanding and modeling of ocean flows, sediment transport, pollutant dispersal and biological processes rely on knowledge of the turbulence characteristics near the ocean bottom.

Several means to measure the turbulence and flow characteristics near the ocean floor have been developed, some involving direct measurements, others relying on an assumed velocity profile or turbulence characteristics. For example, a least squares fit to a measured mean velocity profile can be used to estimate the friction velocity and the bottom roughness length scale, assuming that a logarithmic, constant stress layer exists. This method is sensitive to errors in the height of the sensors (Grant et al. 1984) and in zero offset (Huntley and Hazen 1988). It also requires several velocity sensors with good calibration and stability.

Assuming a balance between turbulent kinetic energy production and dissipation, one can estimate the dissipation from a fit of the Kolmogorov $-5/3$ spectral slope in the inertial range of the vertical velocity spectrum. Using the dissipation and a measured velocity shear, one can then solve for the bottom stress (Grant et al. 1984; Johnson et al. 1994). These approaches depend on the validity of the production-dissipation balance, on estimating spatial spectral levels from temporal spectra, and on the accuracy of estimates of dissipation from the $-5/3$ spectral slope. Alternatively, if data is available at sufficiently small scales, one may estimate the dissipation from a fit of the universal spectrum in the dissipation range (Dewey and Crawford 1988). This
method relies on the existence of sufficient separation between the dissipation range and energy-containing scales.

The most common approach to measuring ocean turbulence is based on acoustic sensors. Acoustic-Doppler Current Profilers (ADCP’s) are the most popular, and have been used very extensively. For example, Lohrmann et al. (1990) and Lu and Lueck (1998) use four-beam instruments to measure 3 velocity components, and Lhermitte and Lemmin (1994) measure 2 velocity components with a three-beam ADCP and fit velocity profiles with logarithmic curves to estimate the shear stress (note, that only part of the velocity distributions actually conform with a logarithmic profile). This technique assumes horizontal homogeneity of the horizontal velocity and the second order moments of the turbulent velocities. Estimates of Reynolds stresses are susceptible to contamination by tilt of the instrument in anisotropic turbulence. Van Haren et al. (1994) use the variance technique on ADCP data to estimate the Reynolds stress, and combine vertical velocity fluctuation measurements with temperature data to estimate the buoyancy flux. They measure the momentum flux, estimate the eddy viscosity, and form an approximate energy balance. Lu and Lueck (1998) discuss the errors and uncertainties of ADCP measurements of turbulence using the variance technique. The Doppler shift of acoustic signal reflection is also employed in the Acoustic Doppler Velocimeter (ADV) for point measurements of 3D velocity components (Kraus et al. 1994; George 1996; Voulgaris and Trowbridge 1998).

The BASS system (Benthic Acoustic Stress Sensor), which is based on acoustic travel time measurements, has been developed to measure the velocity from bottom mounted tripods (Williams et al. 1987; Gross et al. 1994; Williams et al. 1996). Fitting logarithmic profiles to data collected at different elevations can yield estimates of stress (e.g., Johnson et al. 1994;
Trowbridge and Agrawal 1995; Trowbridge et al. 1996). The finite sensor volume of this instrument affects the spatial resolution of the measurement.

Sanford et al. (1999) utilize an advanced electromagnetic velocity and vorticity sensor for measuring fine-scale fluctuations. Sanford and Lien (1999) use this instrument to measure downstream and vertical velocity components as well as fluctuations in a tidal channel, from which Reynolds stress is calculated in 1m vertical bins. The spatial response of this sensor could suffice for resolving the turbulence structure a few meters above the boundary, but very near the bottom, the stress would likely be underestimated. An electromagnetic current meter is also used by Winkel et al. (1996) to provide a reference for acoustic current meter readings.

Laser Doppler Velocimetry (LDV) has been used extensively for measurements of turbulence in laboratories. Oceanic field applications have been infrequent, the primary contributions originating from Agrawal and colleagues (Agrawal and Aubrey 1992; Trowbridge and Agrawal 1995; Agrawal 1996). This point measurement technique can have excellent spatial resolution, but to achieve a small sample volume, one needs to perform the measurements close to the probe. Measurements at longer distances require complex optics or compromises in sample volume size. The LDV data quality also depends on the water transmissivity and on the size of the particles crossing the sample volume.

In a previous paper (Bertuccioli et al. 1999), we have introduced the application of Particle Image Velocimetry (PIV) to oceanic measurements. This method provides an instantaneous distribution of two velocity components over the entire sample area. Various implementation and analysis methods have been used (e.g., Adrian 1991; Grant 1997), but in most cases the fluid is seeded with microscopic tracer particles, and a selected sample area is illuminated with a laser
light sheet. Note, that in the ocean natural seeding is sufficient for obtaining high quality PIV data. To obtain a single data set (i.e., one realization of the spatial velocity distribution within the sample area) the light sheet is pulsed more than once, and the particle traces are recorded on a single frame or on separate frames. Most commonly, the images are divided into small sub-windows and peaks of the correlation function of the intensity distribution yield the mean displacement of all the particles within the window. A sequence of PIV data provides a time series of the spatial distribution in the sample area. Such data can produce information on the entire flow structure, such as velocity profiles, turbulence intensity and shear stress, vorticity distributions, dissipation and turbulent spectra.

In the present paper we present data acquired with the submersible PIV system in the bottom boundary layer off Sandy Hook, NJ in June 1998. We first describe the instrument and the analysis procedures in Section 2. Basic characterization of the flow is presented in Section 3. Velocity spectra are presented in Section 4, followed by a discussion of turbulent kinetic energy dissipation in Section 5. The effects of interpolating velocity distributions and of the interaction of surface waves and turbulence are examined in Sections 6 and 7.

2. Instrumentation and Analysis Procedures

a. Apparatus

A detailed description of the oceanic PIV system can be found in Bertuccioli et al. (1999), and only a brief summary is provided here for completeness. A schematic overview of the surface mounted light source, data acquisition and control subsystems is shown in Figure 1a, and the submersible PIV system is shown in Figure 1b. The light source is a dual head, pulsed dye laser, which provides 2 µs pulses of light at 594 nm, with energy of up to 350 mJ/pulse. The light is
delivered through an optical fiber to a submerged optical probe, where the beam is expanded to a sheet that illuminates the sample area. Based on tests in the lab the maximum energy output at the end of the fiber is 120 mJ/pulse. Images are acquired using a 1024 x 1024 pixel (1008 x 1018 active pixels) Kodak Megaplus-XHF CCD camera, which can acquire up to 15 pairs of frames per second, with essentially unlimited in-pair delay.

The submersible system also contains a Sea-Bird Electronics SeaCat 19-03 CTD, optical transmission and dissolved oxygen content sensors, a ParoScientific Digiquartz, Model 6100A, precision pressure transducer, an Applied Geomechanics, Model 900, biaxial clinometer, and a KVH C100 digital compass. The platform is mounted on a hydraulic scissor-jack to enable acquisition of data at various elevations above the sea floor (the current maximum range is 1.8 m). The platform can also be rotated to align the sample area with the mean flow direction. The flow direction is found by monitoring the orientation of a vane mounted on the platform with a submersible video camera.

**b. Data Acquisition and Analysis**

As noted before, PIV involves illumination of a sample plane with a light sheet and recording of multiple images of the particles within this area. Typically the image is divided into small windows and correlation analysis is used to determine the mean displacement within each window. To obtain a high signal-to-noise ratio we have opted to acquire pairs of images (each exposure on a separate frame) and use cross-correlation for analysis (Keane and Adrian 1992). Details of the image analysis procedures can be found in Roth (1998). Laboratory studies have shown, that an absolute sub-pixel accuracy of about 0.4 pixels can be obtained (which corresponds to a relative accuracy better than 2% if the typical displacement is more than
20 pixels), provided there is a sufficient number of particles per window (5-10), and other requirements involving particle image size and local velocity gradients are satisfied (Dong et al. 1992; Roth et al. 1995; Sridhar and Katz 1995).

With a nominal image size of 1024 x 1024 pixels (active size 1008 x 1018 pixels), an interrogation window size of 64 x 64 pixels and 50% overlap between adjacent windows, each image pair produces a velocity map of 29 x 29 vectors. The magnification is measured directly during each deployment at the test site. For the tests reported here, the measured magnification is 50.9 pixels/cm, yielding a field of view of 20.1 x 20.1cm. Each interrogation window then covers an area of 12.6 mm x 12.6 mm, with a spacing of 6.3 mm between vectors.

Assuming horizontal homogeneity within the sample area, each map provides 29 data points for calculating the velocity at each elevation. For a typical mean velocity of 0.2 m/s, an image-pair acquisition rate of 1 Hz yields a “continuous” record of the velocity components. Our first generation of the data acquisition systems enables acquisition of 130 image pairs, i.e., a data series about two minutes long. As discussed in Section 4.a, we combine this data series to obtain spectra at scales larger than the image size. An advanced Real-Time-Disk system is currently being developed, which can acquire series of up to 35,000 image pairs, namely nearly 10 hours of pairs of 1K x 1K images at a rate of 1 Hz.

c. Deployment

The submersible PIV system was deployed in the New York Bight near the Mud Dump Site, 7 miles east of Sandy Hook, NJ, in June 1998 (Figure 2). The data was taken at station EPA05 (40.42N, 73.86W), within the Expanded Mud Dump area. This region was used until 1977 for dumping dredged material brought from New York by barges, and was later covered with sand to
prevent dispersal of pollutants. The station is close to the summit of an approximately 6km x 3km elevated area, whose long axis is in the North-South direction. The depth near the summit is about 15 m, with the surrounding area being about 25 m deep. The bottom slope at the test site is approximately 3 m/km (SAIC 1995; Schwab et al. 1997).

The bottom composition, surveyed in October 1995, is mostly sandy with no mud or silt. The grain size is $\phi 1-3$ (0.125-0.5mm), with a major mode of $\phi 1-2$ (SAIC 1995). Since no dredged material was added to the site since the date of the survey, we assume that this data is still adequate. The sandy environment is favorable for PIV measurements, since the coarse sand settles quickly, leaving relatively clear water. Visibility measurements done during the experiment indicate light transmission of more than 80% (measured with the SeaCat 25 cm path-length transmissometer). Note, that in another deployment, off Cape May, NJ, we acquired data successfully also when the transmission was less than 50%.

The instrument was submerged from the deck of the R/V Walford, set on a three-point anchor (with the anchors laid 50-100 m from the boat). After it hit the bottom, the platform was raised to the topmost elevation, and was rotated until the vane indicated proper alignment with the mean flow. The same platform orientation was maintained for the entire experiment, at all elevations.

Series of 130 image pairs were obtained at 6 different elevations, with the sample area centered at 20 (i.e., 10-30 cm from the floor), 44, 62, 82, 106, and 128 cm above the bottom. This data spans elevations from 10 cm up to 138 cm above the bottom, covering the range of 0.9 to 1.3 m reported by Huntley (1988) for the thickness of the bottom log layer. Data was first acquired at the highest position, then at the lowermost, and then at gradually increasing elevations. A time delay between elevations of approximately 15 min was required for downloading the data,
changing the elevation and taking pressure readings for determination of the depth of the sample area.

3. Basic Flow Characteristics

a. Mean Flow

During analysis, we first use the clinometer readings to determine the tilt at each measurement station. The differences for the upper 5 stations are at most 0.2° (0.5° downwards at \( z = 62, 82, 106 \) cm, and 0.7° downwards at \( z = 44, 128 \) cm, where \( z \) is the elevation of the center of the sample area), which is the resolution limit of the tilt meter. Only at the bottom position the tilt reading is 0° (a difference of 0.58° in the inclination relative to the average of the other stations - most likely since the tilt of the scissor-jack plate is slightly different when it is fully collapsed). Consequently, the vector maps of the bottom station are rotated to align the whole data set in the same frame of reference.

We do not attempt here to minimize the vertical velocity, \( w \), or its variance, separately for each measurement station, as suggested by Agrawal and Aubrey (1992). The normal velocity in a boundary layer should be exactly zero only at the sea bed (even in a laboratory boundary layer over a flat plate), and different than zero at other elevations. In addition, large-scale bottom slope changes, or external forcing (i.e., horizontal pressure gradients) may also affect the vertical velocity. Furthermore, bottom ripples at scales comparable to the elevation of the measurement point can induce a significant vertical velocity component, which can be of the same order as the mean current. Our data indicates that the vertical velocity diminishes near the bottom (see Figure 4), but does not vanish. Since the entire data set is collected over a period of less than ninety minutes, we opt to maintain the same frame of reference for all the data.
The PIV data enables us to resolve the vertical velocity gradients within the image area, as demonstrated by the sample vector map at the lowest station shown in Figure 3. During analysis, the mean velocity at a given elevation is determined by averaging the instantaneous data for that elevation over the whole series of 130 vector maps. Thus, each point of the vertical distributions of mean velocity, $U(z)$ and $W(z)$ (horizontal and vertical components, respectively), is the average of $29 \times 130 = 3770$ measurements:

$$U(z) = \frac{1}{130 \cdot 29} \sum_{m=1}^{130} \sum_{n=1}^{29} u(x_m, z, t_n)$$

$$W(z) = \frac{1}{130 \cdot 29} \sum_{m=1}^{130} \sum_{n=1}^{29} w(x_m, z, t_n)$$

where $x_m$ and $z$ define location within a certain vector map (comprising 29 x 29 vectors), and $t_n$ indicates maps recorded at different times (130 at each elevation). This procedure is repeated in each of the 29 x 6 vertical positions. The resulting vertical distributions of mean horizontal and vertical velocity components are presented in Figure 4.

The $U(z)$ distributions at the various stations do not form a continuous profile, although their slopes seem continuous. This behavior is most likely caused by time-dependent phenomena with time scales longer than the duration of the data series (130 sec). To illustrate this effect, each data set is divided into 5 subsets, and the mean velocity is calculated for each one. As Figure 5 shows, the mean velocity profile changes within each set, and the characteristics of the variations differ between measurement stations. At $72 < z < 92$ cm, the mean flow accelerates nearly uniformly, with $\partial U/\partial t \approx 0.025$ cm/s$^2$. At $96 < z < 116$ cm, 15 minutes later, the flow first accelerates, then decelerates, and in addition, the vertical distribution of $U(z)$ changes shape.
The time evolution of $U(z)$ and $W(z)$ is also demonstrated in Figure 6, where each point represents an average velocity at a given elevation in one vector map. Each value is obtained by averaging over three rows of vectors (i.e., a “strip” 1.3 cm wide with the center at the desired elevation, a similar “strip” to the one used for the spectra calculations in Section 4). It is evident that the fluctuations in the time series consist of both the effects of turbulence and wave induced motion. As expected, the wave induced orbital motion has more impact on the horizontal velocity (see also Section 6). The irregularity of the fluctuations indicates the presence of large-scale turbulent structures, as well as nonlinearity of the waves and possibly the effect of multidirectional waves with different frequencies.

Near the bottom the vertical distribution of the horizontal velocity is nearly linear (Figure 4). The gradient of $U(z)$ decreases but does not vanish with increasing elevation, as one would expect to find in a boundary layer over a flat surface. This trend indicates the possibility that the flow has mean shear at scales larger than the entire present measurement region. This shear persists for the whole duration of the test, namely, for more than an hour, and is evident at all elevations above the lowest. Moreover, the time series in Figure 6 indicate temporal variations at scales much longer than the duration of the present tests. For example, the mean horizontal velocity at $z = 62$ cm exhibits a distinct negative trend. Such phenomena can only be resolved with measurements spanning a larger range of elevations, possibly the entire water column, and with longer data series. A method to obtain a rough estimate of this large scale forcing is shown in Appendix A.
b. Velocity Fluctuations

To demonstrate the characteristic structure of the turbulence, and the fact that structures can be identified as they are convected between successive realizations, we present in Figure 7a-d sample sequence of four vector maps with distributions of $u(x,z,t)-U(z)$ and $w(x,z,t)-W(z)$. For example, the structures denoted “1” and “2” are convected by about half a frame between realizations, consistent with the mean velocity of about 9 cm/s at $z=20$ cm. The persistence of the structures in the measurement plane at successive times confirms that the out-of-plane velocities are small compared to the in-plane velocity components. In Figure 7e-f, taken from the highest measurement station, small eddies can be identified, which are embedded within larger structures.

We start the analysis of fluctuating velocity fields by decomposing the velocity components $u_i(x_m,z,t_n)$, such that $u_i(x_m,z,t_n) = U_{ei}(x_m,z) + \delta u_{ei}(x_m,z,t_n)$, where $U_{ei}(x_m,z)$ is the ensemble averaged velocity at a point within the vector map, and $\delta u_{ei}(x_m,z,t_n)$ is the deviation from this average (Note, that in this definition the fluctuations contain the effects of both turbulence and wave-induced motion):

$$U_{ei}(x_m,z) = \frac{1}{130} \sum_{n=1}^{130} u_i(x_m,z,t_n)$$

$$u_{ei}'(x_m,z) = \frac{1}{130} \sum_{n=1}^{130} \delta u_{ei}(x_m,z,t_n) = \frac{1}{130} \sum_{n=1}^{130} (u_i(x_m,z,t_n) - U_{ei}(x_m,z))$$

Ensemble averaged correlations between velocity fluctuations are calculated using:

$$u_{ei}' u_{ej}'(x_m,z) = \frac{1}{130} \sum_{n=1}^{130} (u_i(x_m,z,t_n) - U_{ei}(x_m,z))(u_j(x_m,z,t_n) - U_{ej}(x_m,z))$$
where we do not use overbars for the ensemble average for simplicity of notation. Next, we need to consider statistical convergence of the velocity fluctuation data. This is a major concern, since the present measurements provide only 130 realizations at each point in the sample area. Sample running averages of velocity fluctuations and shear stress for a point at the center of the measurement area at the highest elevation \((x_m = 15, z = 128 \, \text{cm})\), and for a point near the bottom \((x_m = 15, z = 12 \, \text{cm})\) are shown in Figure 8. In spite of the small number of realizations at each data point, the quantities of interest exhibit reasonable convergence.

To increase the number of data points included in the averages, we assume horizontal homogeneity, and define RMS velocity fluctuations relative to \(U(z)\) and \(W(z)\), as follows:

\[
\langle u'(z) \rangle = \left( \frac{1}{130 \cdot 29} \sum_{n=1}^{130} \sum_{m=1}^{29} (u(x_m, z, t_n) - U(z))^2 \right)^{1/2} 
\]

\[
\langle w'(z) \rangle = \left( \frac{1}{130 \cdot 29} \sum_{n=1}^{130} \sum_{m=1}^{29} (w(x_m, z, t_n) - W(z))^2 \right)^{1/2} 
\]

\[
-\langle u'w'(z) \rangle = -\frac{1}{130 \cdot 29} \sum_{n=1}^{130} \sum_{m=1}^{29} (u(x_m, z, t_n) - U(z))(w(x_m, z, t_n) - W(z)) 
\]

\(U(z)\) and \(W(z)\) are defined in Eqs. (1) and (2). Note, that \(\langle u' \rangle\) and \(\langle w' \rangle\) are different from \(u_e'\) and \(w_e'\). \(\langle u' \rangle\) and \(\langle w' \rangle\) are obtained from an average of all the data at the same elevation, \(z\), whereas \(u_e'\) and \(w_e'\) are ensemble averages at a given point, over all the vector maps. For convenience we henceforth omit the angled parenthesis <> when referring to averages of data at a given elevation. The convergence trends of the elevation-averaged quantities are similar to those of the point ensemble averages. This is demonstrated in the sample running averages shown in Figure 9 for data at the same elevations as the points in Figure 8.
Vertical distributions of velocity fluctuations and $u'w'$ (evaluated using Eqs. 5–7), as well as an estimate of the velocity fluctuation kinetic energy, are presented in Figures 10 and 11. At this stage we do not attempt to separate wave-induced unsteadiness and turbulence (i.e., the velocity fluctuations include both). For a discussion of the effect of wave contamination, see Section 7. $u'$ and $w'$ are normalized by both $U_{\text{max}} = U(z=115\text{cm}) = 20.8$ cm/s (i.e., the maximum average velocity available from the present data), and $U(z)$, the local average velocity. Vertical distributions obtained from the ensemble-averaged velocity fluctuations, $u_{e}'$ and $w_{e}'$, by averaging for each value of $z$, yield distributions which are almost indistinguishable from those shown in Figures 10 and 11 (the largest discrepancy being about $3\%$), supporting horizontal homogeneity of the turbulence. A comparison of the trends obtained from the present data to those reported for laboratory boundary layer flows is presented in Appendix B.

4. Velocity Fluctuation Spectra

a. Interpolation of vector maps

Each vector map is an instantaneous spatial distribution of the velocity within the sample area. Therefore, up to the scale of a vector map we can calculate spectra based on the measured instantaneous spatial distributions. In order to resolve longer scales, the time series of vector maps is converted into a spatial series using Taylor’s hypothesis. The procedure is introduced in Bertuccioli et al. (1999), but due to its significance to the present analysis, we repeat here the method used to combine the vector maps into an extended composite map.

For each individual vector map we estimate the average horizontal velocity for a 1.3 cm “strip” (3 rows of vectors) around the desired elevation. This average advective velocity is used to evaluate the displacement between successive vector maps. At the lowest elevation ($z = 12$ cm) the displacement between vector maps is 2.6-13.9 cm (132-709 pixels) with an average of 7.3 cm (373 pixels), and there is considerable overlap. At the highest elevations the overlap is very small. At $z = 106$ cm (the elevation with the largest mean current) only $55\%$ of the vector maps
overlap, and the displacement range between 14.2 and 26.4 cm (725-1346 pixels), with an average of 20.3 cm (1035 pixels). In cases where there is a gap between successive vector maps, we need to fill in the missing data using linear interpolation. The typical gap is 1-3 vector spacings, with a maximum of 10 vector spacings. Such gaps occur only for the two highest measurement stations. The effect of the interpolation on the spectra is discussed and demonstrated in Section 6.

The neighboring vector arrays at a specific elevation are interleaved using the appropriate convection velocity to yield an irregularly spaced “spatial” series consisting of $130 \times 29 = 3770$ points, $x_j$. This data is then interpolated onto a regularly spaced array, $X_i$, where the spacing between consecutive points is equal to the spacing between vectors in the individual maps, $s$ (i.e., $s = 0.63$ cm). Each point on the regular array is assigned a velocity $(u^\text{int}(X_i), w^\text{int}(X_i))$, which is the weighted average of the original data points within a range equal to the size of an interrogation window. Thus:

$$
u^\text{int}(X_i) = \frac{\sum (s - |x_j - X_i|) u(x_j)}{\sum (s - |x_j - X_i|)_{|x_j - X_i| < s}}$$

$$w^\text{int}(X_i) = \frac{\sum (s - |x_j - X_i|) w(x_j)}{\sum (s - |x_j - X_i|)_{|x_j - X_i| < s}}$$

A sample case from the $z = 12$ cm data is shown in Figure 12. The region $23.9 \text{ cm} (1216 \text{ pixels}) < x < 26.4 \text{ cm} (1344 \text{ pixels})$ on the irregular grid includes velocities from three successive realizations. Hence, $u^\text{int}(X = 25.1 \text{ cm})$ is the weighted average of the velocities at six points of the original irregular grid, whereas $u^\text{int}(X = 24.5 \text{ cm})$ and $u^\text{int}(X = 25.8 \text{ cm})$ are each the weighted average of five original data points. The total lengths of the patched and interpolated spatial distributions vary with elevation due to the different convection velocities, as summarized in Table 1.
**b. Evaluation of spectra**

To obtain a 1D spectrum, one has to evaluate the Fourier transform:

\[
F_i(k_1) = \sum_n u_i(x_n) W(x_n) \exp(-i k_1 x_n)
\]

(10)

where \(k_1\) is the wavenumber in the “1” direction (in the present data—the horizontal direction in the vector map frame of reference), \(u_i\) is the velocity in the “i” direction at coordinate \(x_n\), and \(W(x_n)\) is a windowing function. The spectral density is then calculated from:

\[
E_{ii}(k_1) = \frac{L}{2\pi N^2} \cdot \sum_n F_i(k_1) F_i^*(k_1)
\]

(11)

where \(L\) is the domain length and \(N\) is the number of points. \(F_i^*\) is the complex conjugate of \(F_i\).

Spectral densities for \(u'\) and \(w'\) (\(E_{ii}(k_1)\) and \(E_{jj}(k_1)\), respectively) are calculated using the regularly spaced interpolated arrays. To take advantage of the entire data without zero padding and still obtain a number of points equal to \(2^n\), each \(u^{int}(X)\) and \(w^{int}(X)\) distribution is divided into two subsets of \(N_{FFT}\) points (values are provided in Table 1). The first subset contains the data at \(X_i\), where \(0 \leq i \leq N_{FFT}-1\) and the second contains the data at \(X_i\), where \(N_{TOT}-N_{FFT} \leq i \leq N_{TOT}-1\).

Three methods have been tested for detrending the data series (Emery and Thomson 1997): Subtraction of the series average, linear detrending (i.e., removing a best-fitted linear curve), and pre-whitening (i.e., creating a series of first differences, whose spectra are converted back to the spectra of the original data). Two windowing functions have been incorporated: a Hanning window and a Cosine Tapered windowing function (applied to the first and last 10% of the data series, while keeping the rest of the data unchanged). The windowed data series is scaled to compensate for energy “loss” due to application of the window. For a continuous form of the windowing function, the scaling coefficient is \(\sqrt{8/3} = 1.633\) for the Hanning window, and 1.084.
for the 10% Cosine Tapered window. For the discrete case, the exact value of the scaling factor depends on the number of points. Note, that the variance of the raw data is conserved only if the data is fully uncorrelated to the window. In the present case, the data is not white, and the length of the data series is not sufficient to ensure decorrelation. Hence, application of a window results in some loss of variance even after scaling. The spectra obtained using different windows and different detrending procedures are very similar, except at the smallest wavenumbers. In this range the spectra are constructed from very few points, hence the associated level of confidence is small. We choose to prewhiten the data and then apply the Cosine tapered window for the patched-interpolated data series. The Hanning window is applied when individual vector maps are considered (since in this case the Cosine-Taper actually introduces a ramp).

Using the two subsets of data and Eqs. (10) and (11), two spectra of $u'$ and $w'$ are evaluated at each elevation. These two spectra are then averaged to yield the spectral densities $E_{11}(k_1,z)$ and $E_{33}(k_1,z)$. In order to increase the amount of data used for each spectrum, we also average data from three successive elevations. Thus, each of the spectra presented in the following is averaged over a 1.3 cm vertical band.

**c. Velocity spectra**

Sample spectra for several elevations are shown in Figure 13. In order to enable a clear view of the trends, the data shown is band-averaged onto a grid of 20 bins per decade. Thus, at the low wavenumbers, the actual data points are plotted, whereas at the highest wavenumbers each point on the plot is an average of 50-200 calculated points. This binning does not affect the trends of the spectra, but it decreases the fluctuations of the plotted curves. The Nasmyth universal spectrum, based on the numerical values given by Oakey (1982) is also presented in Figure 13.
The range of resolved wavenumbers spans about 3 orders of magnitude, except near the floor, where it is slightly smaller. For scales up to the size of a vector map \((k_1 > 31 \text{ rad/m})\), the spectra are derived from spatial distributions that are directly measured, though they are potentially modified by the interpolation (a discussion on the effect of interpolation on the spectra follows in Section 6). The spectra contain only small regions with horizontal tails (that are characteristic of high frequency white noise), at \(k_1 > 400 \text{ rad/m}\), corresponding to wavelengths of less than 1.6 cm.

At the large scales \((k_1 < 8-10)\), \(E_{11} > E_{33}\) at all elevations. These scales contribute most of the velocity variance, which shows a typical ratio of \(u'^2/w'^2 \sim 4\), as shown in Figure 10. The substantially larger energy content of the horizontal velocity fluctuations is due to both the anisotropy of the turbulence and the effect of the wave induced motion (see discussion in Section 7). The anisotropy is evident at very low wavenumbers, where both \(E_{11}\) and \(E_{33}\) seem to reach plateaus at wavenumbers, which are beyond the range of the surface wave spectral peaks, as shown in the Section 7. (Note, that the jaggedness of the spectra in this range is caused by the small number of data points used for evaluating \(E_{ii}\) at very low \(k_1\)).

The difference between \(E_{11}\) and \(E_{33}\) is most significant near the sea floor, and decreases with increasing distance from the bottom. Although \(E_{11}\) changes shape at low wavenumbers (due to the wave effect), the characteristic peak magnitudes remain at the same level, at all the measurement stations. On the other hand, the characteristic peak magnitude of \(E_{33}\) decreases by more than an order of magnitude between \(z = 128 \text{ cm}\) and \(z = 12 \text{ cm}\). Thus, the energy content of the vertical velocity fluctuations is reduced when the distance from the bottom diminishes, whereas the energy content of the horizontal velocity fluctuations is not affected considerably by the elevation. This may indicate the presence of large scale horizontal eddies, whose sizes do not
depend on elevation (as well as the wave contamination). In contrast, the characteristic size of eddies whose axes are horizontal decreases as the floor is approached. This trend, observed in laboratory measurements of flow in turbulent boundary layers, has led to the use of the distance from the wall as an estimate for the integral length scale, $l$, for boundary layer flow. The integral length scale can be estimated from the measured spectra by observing the transition away from the inertial range ($-5/3$) slope at low wavenumbers. For example, at $z = 12$ cm the vertical velocity spectrum flattens at about $k_l \approx 40$ rad/m, i.e., $l \approx 16$ cm (Figure 13), and at $z = 82$ cm the flattening occurs at a wavenumber of about 8, corresponding to $l \approx 80$ cm (Note, that the flattening is partially obscured by the surface wave induced spectral peak).

For small scales $E_{11}$ and $E_{33}$ converge to similar slopes. In some cases $E_{11}$ is slightly smaller than $E_{33}$, and in others the reverse occurs. Least squares fits to the data in the range where it seems to be parallel to a ($-5/3$) slope yield the coefficients $A_{ii}$ shown in Table 2, where $E_{ii} = A_{ii} k_l^{-5/3}$. In all cases the range of wavenumbers where a ($-5/3$) slope line can be fitted is small, and does not exceed a decade. At the lowest station ($z = 12$ cm), this range is even smaller, reducing the confidence level of the line fit. For isotropic turbulence, in the inertial range, $E_{11}(k_l) = 3/4 E_{33}(k_l)$, where $k_l$ is the horizontal wavenumber (Note, that this condition is necessary but not sufficient for isotropy). Clearly, for the present data this ratio is not encountered, indicating that the measured turbulence is not isotropic at inertial range scales either (a discussion of the anisotropy in the dissipation range follows in Section 5).

5. Kinetic Energy Dissipation

Sample “dissipation spectra” (Tennekes and Lumley 1972), i.e. plots of $k_l^2 E_{ii}$, are presented in Figure 14, in both linear and logarithmic scales. The plots of the streamwise fluctuations ($k_l^2 E_{11}$)
do not tail off at high wavenumbers as expected, possibly due to noise (see also Section 6). The plots of $k_1^2 E_{33}$ exhibit clear peaks at $k_1 \approx 100-150$ rad/m (including additional data not shown here), suggesting that the wavelength of peak dissipation is 4-6 cm. Thus, the present data extends to wavenumbers well in the dissipation range.

Several methods to estimate $\varepsilon$, of which three are only possible with 2-D spatial distributions, such as those from PIV data, are described below. The results are discussed in Section 5f.

a. "Direct" estimate of the dissipation tensor

For a Newtonian fluid the dissipation rate is defined as:

$$
\varepsilon = -2\nu S_{ij} S_{ij} = \nu \left( \sum_{ij} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right)
$$

(12)

where $\nu$ is the kinematic viscosity and the indices $i,j$ refer to summation over the three directions.

Using planar PIV we can measure five of the terms in Eq. (12) directly:

$$
\left( \frac{\partial u}{\partial x} \right)^2, \quad \left( \frac{\partial w}{\partial z} \right)^2, \quad \left( \frac{\partial u}{\partial z} \right)^2, \quad \left( \frac{\partial w}{\partial x} \right)^2, \quad \left( \frac{\partial u}{\partial x} \frac{\partial w}{\partial z} \right)
$$

(13)

A similar approach for analyzing PIV data is employed by Fincham et al. (1996).

Using continuity we obtain:

$$
\left( \frac{\partial v}{\partial y} \right)^2 = \left( -\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z} \right)^2 = \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 + 2\left( \frac{\partial u}{\partial x} \frac{\partial w}{\partial z} \right)
$$

(14)
For isotropic turbulence, the terms in Eqs. (13) and (14) constitute 8/15 of the total dissipation. Additional terms are estimated, assuming that all lateral fluctuations have similar average magnitudes:

\[
\left( \frac{\partial u}{\partial y} \right)^2 = \left( \frac{\partial w}{\partial y} \right)^2 = \left( \frac{\partial v}{\partial x} \right)^2 = \left( \frac{\partial v}{\partial z} \right)^2 = \frac{1}{2} \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2
\]

(15)

Introducing into Eq. (12), we obtain the first estimate of the dissipation rate, \( \varepsilon_D \):

\[
\varepsilon_D = 3N \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial w}{\partial x} \right)^2 + 2 \left( \frac{\partial u}{\partial z} \right)^2 + \frac{2}{3} \left( \frac{\partial u}{\partial x} \right)^2 \right]
\]

(16)

Note, that in this case, the only assumption is that the cross-stream gradients have the same average magnitude as the measured in-plane cross gradients (this assumption is supported by the similar magnitudes of the measured gradients, \( (\partial u/\partial z)^2 \) and \( (\partial w/\partial x)^2 \)). In isotropic turbulence these assumptions become identities. In addition, all the directly measured gradients are used in the estimate. Thus, the assumptions leading to Eq. (16) are weaker than those involved in isotropy based methods, such as the line fit and dissipation integral described in the following two sections.

**b. Line fit in the inertial range**

Using an assumption of isotropic, homogeneous turbulence, at least in the inertial range (which is not accurate for the present data), the dissipation rate can be estimated from (Hinze 1975; Tennekes and Lumley 1972):
The streamwise velocity spectra are more susceptible to contamination by surface waves (see Section 7), which can leak to wavenumbers within the inertial range. Therefore, we evaluate the dissipation rate, $\varepsilon_{LF}$, based on the isotropic ratio of $E_{11} = \frac{1}{4} E_{33}$, and use the values of $A_{33}$ from Table 2. Similar estimates of $\varepsilon_{LF}$ are obtained by fitting the Nasmyth universal spectrum (Oakey 1982) in the inertial range only. The method is essentially similar to that proposed by Stewart and Grant (1962), and has also been used extensively for estimating the dissipation rate in turbulent laboratory flows.

\textbf{c. Integral of the dissipation spectrum}

For homogeneous, isotropic turbulence, the dissipation rate is:

$$\varepsilon = 15\nu \left( \frac{\partial u}{\partial x} \right)^2 = \frac{15}{2} \nu \left( \frac{\partial v}{\partial x} \right)^2$$

(18)

The Fourier representation of (18) can be used to estimate the dissipation rate from the one-dimensional spectrum of vertical velocity fluctuations (Monin and Yaglom 1975):

$$\varepsilon_{DS} = \frac{15}{2} \nu \int_0^\infty k_1^2 E_{33}(k_1) dk_1$$

(19)

The present data does not extend to the Kolmogorov scale. Moreover, at the largest wavenumbers there is some indication of noise in the dissipation spectra. Therefore, the area under the measured dissipation spectra represents only part of the total dissipation. To estimate the portion of $\varepsilon$ in the range of $k_1$ that we integrate, we use the Nasmyth universal spectrum. We first determine a cutoff wavenumber, which constitutes the upper limit of the integral (this cutoff should be large enough to avoid loss of data and small enough to exclude noise). The universal
spectrum is fitted to the measured data, such that the integrals of the universal spectrum and of the measured data are equal in the prescribed wavenumber range, Figure 15. This fit yields the total dissipation (which parameterizes the universal spectrum). Thus, $\varepsilon_{DS}$ is evaluated by extending the measured dissipation spectrum integral based on the universal spectrum. For the present data, dissipation estimates obtained for two cut-off wavenumbers ($k_1 = 300$ rad/m, and $k_1 = 470$ rad/m) yield similar values (within 10%), except at the bottom station, where the difference is about 30%. This discrepancy at the lowest station can be attributed to the increasing impact of the wave induced motion: At this elevation the low local convection velocity affects the location of the wave induced spectral peak observed at $k_1 \approx 80$ rad/m (see Section 7). This not only moves the wave induced spectral peak closer to the peak dissipation wavenumber, but also increases the contribution of wave contamination (since dissipation scales with $k_1^2$). A lower cut-off increases the relative weight of this contamination. On the other hand, high cut-offs may introduce noisy wavenumbers into the estimate. The values shown in Table 3 are calculated with the cut-off at $k_1 = 300$ rad/m.

**d. Locally axisymmetric turbulence**

Considerable experimental data indicates that assumptions of local isotropy do not hold in a variety of flow situations. An alternative approach (e.g., George and Hussein 1991; Antonia et al. 1991) hypothesizes that the turbulence is locally axisymmetric, namely, invariant to rotations around a preferred axis (the streamwise direction in the present case). This assumption is less stringent than the requirements for isotropy, and leads to the following relationships:
\[
\left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial z}\right)^2 ; \left(\frac{\partial v}{\partial x}\right)^2 = \left(\frac{\partial w}{\partial x}\right)^2 ; \left(\frac{\partial v}{\partial z}\right)^2 = \left(\frac{\partial w}{\partial y}\right)^2
\]

\[
\left(\frac{\partial v}{\partial y}\right)^2 = \frac{1}{3}\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right)
\]

\[
\frac{\partial v}{\partial z} \frac{\partial w}{\partial y} = \frac{1}{6}\left(\left(\frac{\partial u}{\partial x}\right)^2 - 2\left(\frac{\partial v}{\partial z}\right)^2\right)
\]

\[
\left(\frac{\partial u}{\partial y} \frac{\partial v}{\partial x}\right) = \left(\frac{\partial u}{\partial z} \frac{\partial w}{\partial x}\right) = -\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2
\]

The dissipation rate can then be estimated from gradients in the measurement plane:

\[
\varepsilon_{AS} = v\left[-\left(\frac{\partial u}{\partial x}\right)^2 + 8\left(\frac{\partial w}{\partial z}\right)^2 + 2\left(\frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{\partial w}{\partial y}\right)^2\right]
\]  

(21)

e. Energy flux across equilibrium range

Large Eddy Simulation (LES) of turbulent flows is based on filtered Navier-Stokes equations (e.g., Rogallo and Moin 1984; Lesieur and Metais 1996):

\[
\frac{\partial \tilde{u}_i}{\partial t} + \tilde{u}_j \frac{\partial \tilde{u}_i}{\partial x_j} = -\frac{\partial}{\partial x_j}\left(\frac{\tilde{p}}{\rho} \delta_{ij} + \tau_{ij}\right) + \nu \frac{\partial^2 \tilde{u}_i}{\partial x_j \partial x_j}
\]  

(22)

where the tilde denotes a spatial filtering operation at a scale $\Delta$, and the subgrid-scale (SGS) stress tensor is defined as

\[
\tau_{ij} = u_i \tilde{u}_j - \tilde{u}_i \tilde{u}_j
\]  

(23)

If the scale of the filter corresponds to wavelengths within the inertial range, the condition of equilibrium suggests that the energy flux to smaller (subgrid) scales should provide an estimate of the dissipation rate (Liu et al. 1994, 1999). Thus:

\[
\varepsilon_{SG} = -\tau_{ij} \tilde{S}_{ij}
\]  

(24)
where $\tilde{S}_{ij} = 1/2 \left( \partial \tilde{u}_i / \partial x_j + \partial \tilde{u}_j / \partial x_i \right)$ is the filtered rate of strain tensor. Since the planar PIV provides only part of the tensor terms, we approximate the others, similar to the approach in Section 5.a. Inserting the known terms and the approximated ones

$$
\tau_{12} \tilde{S}_{12} = \tau_{23} \tilde{S}_{23} = \tau_{13} \tilde{S}_{13}
$$

$$
\tau_{22} \tilde{S}_{12} = \frac{1}{2} \left( \tau_{11} \tilde{S}_{11} + \tau_{33} \tilde{S}_{33} \right)
$$

into Eq. (24) yields the subgrid-scale energy flux estimate of the dissipation rate:

$$
\varepsilon_{SG} = \frac{1}{2} \left( 3 \tau_{11} \tilde{S}_{11} + 3 \tau_{33} \tilde{S}_{33} + 12 \tau_{13} \tilde{S}_{13} \right)
$$

Note, that this estimate is based on a third-order moment of the velocity, while all the previous methods are based on second-order moments. Since it is calculated from filtered data, high wavenumber noise is removed.

We need to ensure that the spatial filtering is done at a scale within the inertial range. At the same time, we need to minimize this scale in order to have as many data points on the filtered grid as possible. We set the filter size $\Delta$ to 8 vector spacings (corresponding to a wavenumber of $k_1 = 124$ rad/m). While this value may be somewhat too low, it is the largest one that allows us to have a filtered array of 3 x 3 vectors from each original vector map. The local filtered rates of strain are evaluated by offsetting filtered grids by one vector spacing and differencing the offset filtered velocities.

**f. Discussion of dissipation estimates**

The kinetic energy dissipation rate estimates provided by the various methods are presented in Table 3 and Figure 16. Table 3 also includes the Kolmogorov length scale $\eta$ for the “direct” estimates, which is calculated using the kinematic viscosity of seawater for the temperature at the test site ($9^\circ$C), $\nu = 1.4 \times 10^{-6}$ m$^2$/s (Kennish 1989):
\[ \eta = \left( \frac{v^3}{\varepsilon} \right)^{\frac{4}{3}} \]  

(27)

All the methods provide the same order-of-magnitude estimates for the dissipation rate. The largest discrepancy, between \( \varepsilon_{SG} \) and \( \varepsilon_{AS} \) is observed at the bottom station (106\%), while at \( z = 44 \) cm the discrepancy between the highest value (\( \varepsilon_{LF} \)) and the lowest estimate (\( \varepsilon_{SG} \)) is only 32\% (Note the reverse trend, where \( \varepsilon_{SG} \) provides the largest estimate at \( z = 12 \) cm, and the lowest one at \( z = 44 \) cm). The present dissipation rate estimates are within the range of typical values. For example, they are comparable to values reported by Simpson et al. (1996) for a well-mixed boundary layer in the Irish Sea, Gross et al. (1994), off the California coast, as well as the data of Gargett et al. 1984. The present \( \varepsilon \) estimates are somewhat larger than some values obtained in coastal waters (e.g., Stewart and Grant 1962; Oakey 1982; Dewey and Crawford 1988; Gargett 1994; Lueck et al. 1997), but are smaller than values measured in a tidal bottom boundary layer (Sanford and Lien 1999), in a tidal channel (Gargett and Moum 1995), and over a rippled bed (Agrawal and Aubrey 1992).

The best dissipation rate estimate is provided by the “direct” method (\( \varepsilon_D \)), since it involves the least stringent assumptions. In addition, it is based on the original data contained in the PIV images, without additional processing (such as interpolation or computation of spectral estimates, which are required for \( \varepsilon_{LF} \) and \( \varepsilon_{DS} \)). The locally axisymmetric turbulence dissipation estimates, \( \varepsilon_{AS} \), are very close to \( \varepsilon_D \), as expected. Nonetheless, the \( \varepsilon_{AS} \) and \( \varepsilon_D \) values are slightly different since not all the available measured gradients are taken into account in \( \varepsilon_{AS} \).

Estimation of dissipation using a third-order moment (\( \varepsilon_{SG} \)) requires a large number of data points to converge. For example, more than 40000 points are required for convergence in the jet data of
Liu et al. (1994). In the present case we only have a total of 1170 points, which reduces the statistical confidence. The $\varepsilon_{SG}$ values therefore show the largest discrepancies relative to the other estimates.

The line-fit and dissipation integral methods, $\varepsilon_{LF}$ and $\varepsilon_{DS}$, respectively, yield results which are generally higher than $\varepsilon_D$. This trend agrees with the data of Fincham et al. (1996), who find that the assumption of isotropy leads to an overprediction of the dissipation rate by as much as 375% for stratified grid turbulence. The estimates from the integral of the dissipation spectrum ($\varepsilon_{DS}$) are closer to $\varepsilon_D$, since they are based on the whole range of available data, whereas the (-5/3) line-fit estimate are only based on a portion of the available spectrum. Also, the relative effect of contamination by wave-induced motion is probably larger for $\varepsilon_{LF}$, since it is based on a range of wavenumbers which is closer to the equivalent scale of the waves (see Section 7). Even though the dissipation integral is also contaminated by the waves, the major contribution to its value comes from much smaller scales.

It is reassuring, that despite the numerous assumptions involved in some of the estimation methods, all of them reproduce the trends of the dissipation rates at the various stations. Note, that the profile shown in Figure 16 actually represents both spatial and temporal variations due to the time delay between measurements. The dissipation rate is nearly uniform (within 33%) at all elevations, except for $z = 106$ cm. At this station, the dissipation is nearly twice as large, which also coincides with the maximum large-scale forcing and the largest values of the mean current (see Section 3a).
6. Impact of Vector Map Interpolation on Spectra

As described in Section 4a, the individual vector maps are “patched” to produce an extended vector map, based on Taylor’s hypothesis. In order to examine the impact of this procedure on the spectra, we compute spatial spectra directly from the spatial distributions of the 130 vector maps at a given elevation, and average them (denoted “IM” in the following). The resulting average spatial spectra are compared to the spectra obtained from the corresponding extended interpolated data series (denoted “ES”). Sample comparisons of the velocity spectra and of the dissipation spectra are shown in Figures 17 and 18, respectively. The spectra obtained using the two methods are similar in the low wavenumber part of the overlapping range, whereas at large wavenumbers, the extended series contains less energy (this trend is also observed in the data of Bertuccioli et al. 1999). Note, that the “IM” spectra are not band-averaged, as each individual spectrum contains only 16 points in wavenumber space. Rather, they are smooth since they are calculated from the average of 130 individual spectra.

Estimates of the total energy content in the range of overlapping wavenumbers are calculated using Eq. (28), and are presented in Table 4.

\[ e_i = \int E_i dk_i, \quad i = 1,3 \quad (28) \]

The IM vertical velocity spectra contain more energy than the ES spectra, with the largest difference in \( E_{33} \) occurring near the bottom: 55%. The IM horizontal velocity spectra contain more energy than the corresponding ES spectra at the low elevations, but at the high elevations the ratio is reversed, and the \( E_{11} \) spectra of the extended series contain more energy. Two effects contribute to these trends: Firstly, at low elevations, there is considerable overlap between vector maps, resulting in some filtering of the high wavenumber energy due to the interpolation scheme.
The overlap (and hence the interpolation induced smoothing) is reduced away from the bottom. The second effect is due to wave induced motion, which has substantial effect on the spectrum of the extended data series. The characteristic time scales of the waves and of the energy containing eddies are similar. Hence, in the extended series, which is constructed using Taylor’s hypothesis, the wave induced motion appears at equivalent wavenumbers (see Section 7) of 2-10 rad/m, and leaks to higher harmonics, which are within the range where we compare the IM and the ES spectra (i.e., $k_1 = 31$-500 rad/m). The IM spectra, on the other hand, are based on measured spatial distributions within individual vector maps. There is a significant difference between the spatial scale of the measurement area (20 cm) and the wavelength of the surface waves (e.g., in 15 m deep water, a wave with a 7.5 s period has a wavelength of 75 m). Due to this large separation of scales in the spatial domain, the IM spectra are not affected by the wave induced motion. The wave effect on the horizontal velocity is more significant than on the vertical velocity (see Section 7), and it is reduced as the bottom is approached. Hence, it introduces more energy into the ES spectra of $E_{11}$ at the high elevations (note, that the largest difference between the ES and IM horizontal spectra is observed at $z = 82$ cm, where the wave spectral peak is most pronounced). The $E_{33}$ spectra are influenced mostly by the smoothing, which is most significant near the bottom.

The shape of the ES dissipation spectra in Figure 18 follows the universal spectrum more closely than the IM spectra, which exhibit a roll-off at $k_1 = 200$-300 rad/m. This points to the possibility of noise contamination in the IM spectra at large wavenumbers. The interpolation involved in the generation of the ES vector maps may dampen such noise, and may actually provide a better representation of the average spectral energy density.
Most of the deviation between the two sets of spectra is evident in the high wavenumber region. Thus, dissipation estimates calculated with a line fit of a $k^{-5/3}$ curve to the inertial range (Eq. 17) are expected to be in closer agreement than those evaluated from an integral of the dissipation spectrum (Eq. 19). The procedures outlined in Sections 5b and 5c are repeated for the IM spectra. As listed in Table 5, best-fitting $k^{-5/3}$ curves to the IM spectra in the inertial range yields dissipation rate estimates which are within 20% of the ES estimates. Only at the lowest station, at $z = 12$ cm, is the discrepancy considerably larger (37%). At this elevation the range with a (-5/3) slope is extremely small, and the accuracy of the slope-based estimates is low. The dissipation estimates based on the integral of $k_i^2E_{II}(k_i)$ IM spectra are 29%-122% higher than the corresponding ES estimates, as expected. Once again, the largest discrepancy is near the bottom. All the IM dissipation estimates yield larger values than the “direct” ones ($\epsilon_D$).

The IM velocity spectra have a “hump” in $E_{II}(k_i)$ at approximately $k_i = 200$ rad/m, where the values of the streamwise spectral density become larger than those of the spanwise spectra. The ES spectra also show a similar behavior, but the “hump” is less pronounced. This trend is not an artefact of the detrending or windowing procedures, since it is observed even when no windowing or detrending is applied.

In addition to the spectra in the horizontal ($x$ or $k_1$) direction, PIV data allows us to calculate spectra of velocity fluctuations in the vertical ($z$ or $k_3$) direction. As shown in Figure 19, the distributions of $E_{II}(k_1)$ are similar to those of $E_{33}(k_3)$, and the trends of $E_{33}(k_1)$ are similar to those of $E_{II}(k_3)$. Thus, both $E_{II}(k_1)$ and $E_{33}(k_3)$ have the “hump” described earlier, indicating that this phenomenon is independent of the direction relative to the mean flow. At this stage we do not have an explanation for this trend. One possibility may be the “bottleneck effect” (e.g., Falkovich 1994; Lohse and Muller-Groeling 1995), which is also observed experimentally in
high Reynolds number boundary layer flow (Saddoughi and Veeravalli 1994). Another could be narrow-band noise caused by an as yet unidentified source. Note, that Voulgaris and Trowbridge (1998) report similar trends with ADV data, which they attribute to effects of viscous dissipation, production, attenuation due to spectral averaging over the ADV sample volume, and measurement noise.

7. Wave-Turbulence Interaction

In the following section we address the issue of interaction of waves and turbulence and its effect on the turbulence spectra. The surface waves induce unsteady velocity components at temporal scales which are comparable to the characteristic turbulence time scales although the spatial scale of the waves is much larger than the characteristic turbulence length scales. Since the power spectra that we calculate are based on Taylor’s hypothesis for spatial scales larger than a single vector map, the signatures of the surface waves alter the shape of the spectra significantly.

Pressure variations, which occur due to the motion of surface waves, are recorded simultaneously with the PIV data, at a rate of 7 Hz. We compute the frequency-domain power spectrum of the pressure data using FFT’s. The pressure spectrum for each measurement station is translated to equivalent wavenumbers using the mean convection velocity at that elevation:

$$
 k = \frac{2\pi}{U/f_w}
$$

where $f_w$ is the frequency of the wave.

Figure 20 shows three sample pressure spectra together with the velocity spectra. It is evident that there are several surface wave peaks. The highest pressure spectrum peak, which is also the
one that exists in all the data (including additional results not shown here) appears to have a period of about 7.5 s. Since the convection velocity increases with elevation, the spectral peaks are shifted to lower wavenumbers with increasing elevation (at \( z = 12 \text{ cm} \) the peak is at \( k_l = 10.3 \text{ rad/m} \), whereas at \( z = 1.2 \text{ m} \) the peak is at \( k_l = 3.9 \text{ rad/m} \)). Some of the peaks of the pressure spectra coincide with peaks in the horizontal velocity spectra. This agreement indicates that the shape of the velocity spectra in that range is affected significantly by the wave motion, in addition to the large scale turbulence.

According to linear wave theory (Dean and Dalrymple 1984):

\[
\begin{align*}
    u_w &= \eta_w \sigma \frac{\cosh k_w (h + z)}{\cosh k_w h} \\
    w_w &= \eta_w \sigma \tan \sigma t \frac{\sinh k_w (h + z)}{\sinh k_w h}
\end{align*}
\]

(30) \hspace{1cm} (31)

where \( u_w \) and \( w_w \) are the horizontal and vertical wave induced velocities, respectively, \( h \) is the mean depth, \( z \) is the distance below the mean surface (\( z = 0 \) at the surface, \( z = -h \) at the bottom), \( \eta_w \) is the surface displacement given by \( \eta_W = \eta_0 \cos (k_w x - \sigma t) \), and the angular frequency \( \sigma \) is given by the dispersion relation \( \sigma = U k_w + \sqrt{g k_w \tanh k_w h} \). Eqs. (30) and (31) indicate that the wave frequency affects the amplitude of the induced velocity. Thus, two pressure peaks with the same magnitude, but at different \( k_w \), and therefore different \( \sigma \) will have a different effect on the velocity. For example, in Figure 20a the \( u \) spectral peak associated with the pressure at \( k_l = 2.1 \text{ rad/m} \) is lower than the corresponding peak at \( k_l = 4.2 \text{ rad/m} \), although the relative magnitude of the pressure peaks is the opposite. Note, however, that this trend may also be caused by a wave propagating out-of-plane at an angle to the \( x \) axis of our interrogation window.
Such a wave would have a reduced effect on the measured velocity, but the effect on the pressure spectrum would be the same as that of a wave parallel to the measurement plane.

The wave effect on the vertical velocity depends on $\sinh k_W (h+z)$, and is therefore significantly weaker, as demonstrated in all the sample results. Similar trends for both $u$ and $w$ are reported by Agrawal and Aubrey (1992). In the past, researchers (Grant et al. 1984; Dewey and Crawford 1988; Huntley and Hazen 1988; Agrawal et al. 1992; Gargett 1994; Gross et al. 1994) have used the vertical velocity spectra to calculate the turbulence parameters, since they are less affected by the waves. This approach is also adopted for the dissipation evaluations in Section 5, because the wave effect on $w$ is weak for all the present data, although it does increase slightly with elevation.

In the analysis presented here we do not attempt to separate wave-induced unsteadiness and turbulence. The procedure proposed by Trowbridge (1998) to remove the wave contamination from the turbulence data cannot be applied to the present data, since it requires data from points separated by more than the characteristic turbulence length scale. The present data provides measured velocities at locations separated by no more than 20 cm, which does not satisfy this requirement. In future deployments, we plan to measure spatial correlations spanning larger scales, most probably using more than one camera simultaneously. For example, two cameras, each with a sample area of 50 x 50 cm, located with their centers 1 m apart, can provide (using combined spatial correlation within each image and cross-correlation between images) true spatial spectra at scales up to 1.5 m, i.e., $k_f = 4.2 \text{ rad/m}$. For the present data, this scale is beyond the wave spectral peak. In a sense, such use of two PIV images performs a similar function to the procedure proposed by Trowbridge (1998).
Similarly, we do not apply the pressure-velocity cross-correlation filtering method proposed by Agrawal and Aubrey (1992), since the present time series includes only 130 data points. Filtration of the wave effect will be attempted with longer data series, which will contain significantly more data points and thus can allow the velocity-pressure cross-correlation to converge. Rather than attempt a crude approach to isolating turbulence from wave induced motion, we prefer to defer this analysis to data sets collected specifically to address this issue.

8. Concluding Remarks

Particle Image Velocimetry (PIV) provides two-dimensional velocity distributions within a prescribed sample area. While the technique is well established in laboratory studies, the present apparatus constitutes, to the best of our knowledge, the first attempt to implement it in an oceanic environment. Various tests, and in particular the latest deployment of the system in the New York Bight have proved its feasibility and its ability to provide high quality data.

The two-dimensional velocity distributions available with PIV enable evaluation of spectra which are based on true spatial distributions. The data also provides measured spatial velocity gradients, which enables us to test some of the commonly used assumptions regarding the flow structure, and in particular isotropy. For instance, the accuracy of dissipation estimates obtained from spatial spectra derived from temporal data relies on the validity of isotropy assumptions, as well as Taylor’s hypothesis, and on the existence of an equilibrium inertial range.

While the large scale turbulence in a boundary layer is necessarily anisotropic, it is common to regard the small-scale turbulence as locally isotropic and employ various forms of the Kolmogorov model. The present data clearly show departures from isotropy at all scales. In the inertial range, $E_{11}$ is slightly higher than $E_{33}$, and at dissipation scales it is considerably higher,
whereas isotropy requires that $E_{11}$ be smaller than $E_{33}$ (Note, that the small-scale portion of the spectra is based on measured spatial distributions).

The measured turbulence is anisotropic, even at the smallest resolved scales, within the dissipation range. There is evidence that local isotropy may be approached away from walls (e.g., Gargett et al. 1984), but it is not observed close to a surface (Antonia et al. 1991). Durbin and Speziale (1991) show that exact local isotropy cannot exist in the presence of mean shear. The existence of anisotropy at the dissipation as well as the inertial scales is shown experimentally by Garg and Warhaft (1998) for a homogeneous shear flow in a wind tunnel. Saddoughi and Veeravalli (1994), for high Reynolds number flow, and Antonia and Kim (1994), for low Reynolds number flow, present criteria for a maximum strain rate which allows viewing the local turbulence as nearly isotropic. The values of the strain-rate in the present data are near this limit or slightly above it. The dynamics of the small scales can also be affected by non-local interactions, where large-scale anisotropy directly affects the smallest scales (Brasseur and Wei 1994; Zhou et al. 1996). This effect is more significant as the scale separation increases. Thus, it is not surprising that the present data does not fulfill requirements for isotropy even at dissipation scales.

The integral length scale associated with the vertical velocity fluctuations should scale with distance from the bottom. This trend is clearly demonstrated in the present data, where $E_{33}$ decays with decreasing elevation. As expected, the vertical velocity fluctuations are damped more than the horizontal fluctuations, leading to anisotropy at the largest resolved scales. Although the data at these scales is contaminated by the surface waves, the velocity spectra extend to scales that are larger than those corresponding to the wave spectral peaks. Even at the
highest elevation (128 cm above the bottom), the energy content of the horizontal fluctuations at the lowest resolved wavenumbers is larger than the vertical.

In spite of the measured departure from the simplifying assumptions of homogeneity and particularly isotropy, some general trends derived from these assumptions are supported by the present data: Dissipation estimates obtained using various methods, part of which rely on assumptions of isotropy, provide same order-of-magnitude values. Comparison with estimates based on measured terms of the dissipation tensor (provided by the 2-D data) yields differences of less than 40%. In addition, the assumption of horizontal homogeneity of the turbulence seems reasonable and introduces only small errors in the turbulence characteristics.

Acknowledgments

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APPENDIX A

Estimate of Large Scale Forcing

An estimate of the external forcing for the flow in the boundary layer can be obtained directly from the averaged two-dimensional PIV data. We write the ensemble averaged horizontal momentum equation:

\[
\frac{\partial U_e}{\partial t} + U_e \frac{\partial U_e}{\partial x} + V_e \frac{\partial U_e}{\partial y} + W_e \frac{\partial U_e}{\partial z} - f V_e = -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{\partial (u_e' u_e' \rho)}{\partial x} - \frac{\partial (u_e' v_e' \rho)}{\partial y} - \frac{\partial (u_e' w_e' \rho)}{\partial z} + \nu \frac{\partial^2 U_e}{\partial z^2}
\]

(A1)

where \( f \) is the Coriolis parameter, \( p \) is the pressure, \( \rho \) is the water density, and \( \nu \) is the kinematic viscosity. \( \frac{\partial U_e}{\partial t} \) refers to time dependent variations at scales longer than the present data acquisition time, which can be measured directly in longer time series. Away from the bottom the gradients of the velocity fluctuation correlations, \( \partial/\partial x \left( u_e' u_e' \right) \), as well as the viscous term, are small (this assumption is tested later). Since the sample area is aligned with the mean flow, \( V_e \ll U_e \), and we also assume that out-of-plane gradients of the mean velocity are small, hence we neglect \( V_e \frac{\partial U_e}{\partial y} \).

Using the continuity equation, the remaining convective terms can be rewritten as:

\[
U_e \frac{\partial U_e}{\partial x} + W_e \frac{\partial U_e}{\partial z} = -U_e \frac{\partial W_e}{\partial z} + W_e \frac{\partial U_e}{\partial z} = -U_e \frac{\partial}{\partial z} \left( \frac{W_e}{U_e} \right)
\]

(A2)

Thus, Eq. (A1) is reduced to:

\[
\frac{\partial U_e}{\partial t} - U_e^2 \frac{\partial}{\partial z} \left( \frac{W_e}{U_e} \right) \approx -\frac{1}{\rho} \frac{\partial \rho}{\partial x} + f \bar{V}
\]

(A3)
Equation (A3) indicates, that the sum of the vertical gradient of \( W_e / U_e \) and the unsteady term provides an estimate for the large scale forcing of the local flow.

The data in Figure 5 suggests a typical value of \( \partial U_e / \partial t \sim 0.04 \text{cm/s}^2 \), much smaller than the magnitude of \( -U_e^2 \partial (W_e / U_e) / \partial z \) for the present conditions. Thus, only the vertical gradient term dominates the left-hand-side of Eq. (A3), and the resulting estimates of the large-scale forcing for the six measurement stations are presented in Figure A1.

The error introduced by neglecting the unsteady term is less than 10% of the estimated forcing, at most stations. The maximum temporal rate of change according to the estimates from the data in Figure 5 is about 0.16 cm/s\(^2\) (25% of the large scale forcing estimate), at 52 < \( z < \) 72 cm. Note, however, that such estimates may only capture part of the unsteadiness occurring over longer time spans, since each subset in Figure 5 represents only 26 s of data.

The measured vertical distribution of \( u'_e w'_e \), and the measured horizontal distribution of \( u'_e u'_e \) are shown in Figure A2. The maximum magnitude of \( \partial (u'_e w'_e) / \partial z \) is about 0.05 cm/s\(^2\), at the highest elevation (Figure A2a). The maximum value of \( \partial (u'_e u'_e) / \partial x \) is about 0.2 cm/s\(^2\), at the lowest elevation (Figure A2b), where gradients of stresses are expected to be larger due to the proximity to the bottom, and the applicability of the large-scale forcing estimate is reduced. At other elevations this gradient is considerably smaller. \( \partial (u'_e v'_e) / \partial y \) is not measured, and we assume that its magnitude is not larger than the other gradients. Hence, neglecting \( \partial (u'_e u'_j) / \partial x_j \) in Eq. (A1) may introduce errors of no more than 50% in the estimated large-scale forcing. Consequently, the data in Figure A1 provides order-of-magnitude estimates of the large scale forcing, which drives the flow in the boundary layer.
The results indicate that the large scale forcing changes substantially between measurements. Most notably, it reverses direction between 12:33 and 12:46. Since no measured tide data is available for the test site (40.42°N, 73.86°W) at the time of the experiment, we use “Tides and Currents v.2” (Nautical Software, Inc.) to estimate the variations of the tidal current at the nearest available locations. North of our site, at Ambrose Channel, NY (40.52N, 73.97W), high tide is predicted at 1:03pm, while south of our site, at Long Branch, NJ (40.30N, 73.97W) high tide is predicted at 10:50am. The tide model suggests that the direction change of the large scale forcing is related to the tidal cycle.

The estimated magnitude of the forcing is larger than the typical values encountered in ocean flows. However, our estimates are based on fairly strong assumptions, which could affect the numerical values of the results. Nonetheless, the trends are consistent with other data (such as the tide reversal, and the time evolution of the mean current shown in Figure 5). Other possible causes may be the effect of unresolved large scale structures, for example, very low frequency propagating waves, such as infragravity waves (Herbers et al. 1994, 1995), or the proximity to the large estuaries which influence the New York Bight region.

**APPENDIX B**

**Comparison to Laboratory Boundary Layer Data**

The present data includes the effect of wave induced motion, in addition to turbulence, as discussed in Section 7. Although we do not attempt here to separate the two effects, comparison of the measured velocity fluctuations to those observed in boundary layer flows in laboratories reveals similar trends.
The vertical distributions of velocity fluctuations, shown in Figure 10, indicate that the magnitudes of the fluctuations are higher than those measured in laboratory flows over flat plates (e.g., Hinze 1975). This is expected, since they also include the wave contamination. Both $u'$ and $w'$ decrease slightly as the bottom is approached. $u'/U_{\text{max}}$ ranges from about 0.15 at the highest station to about 0.12 near the floor, and $w'/U_{\text{max}}$ decreases from about 0.08 to 0.05 (the data for $98<z<118$ cm deviate from this general trend). In laboratory boundary layer flows over smooth flat plates, such a reduction of the turbulence intensities is expected only at $z/\delta<0.2$ for $w'$ and $z/\delta<0.01$ for $u'$ (Hinze 1975). Note also, that the present data is taken at the bottom of the coastal ocean, which has significant roughness. Experimental and numerical data for flow over a rough wall (at relatively low Reynolds numbers) indeed show that the magnitudes of the Reynolds stresses (both normal and shear) are larger than those encountered in flows over smooth walls (e.g., Krogstad et al. 1992; Shafi and Antonia 1997).

When the velocity fluctuations are normalized by the local mean velocity, the relative intensity of the horizontal component increases near the bottom to about 0.4 at $z = 10$ cm. This trend is in qualitative agreement with flat plate boundary layer data, but again the present magnitude is higher. $w'/U(z)$ also increases near the bottom, and its magnitude is higher than the laboratory data. As discussed in Section 7, the anisotropy shown in Figure 10 ($u'/w' \sim 2$ at all elevations) is caused primarily by both the large scale turbulence and the wave induced unsteadiness. The small scale turbulence (e.g., up to the scale of a vector map) is closer to being isotropic, but is still anisotropic.

The velocity fluctuation kinetic energy, $q^2$, whose vertical distribution is shown in Figure 10 together $-u'w'$ is estimated as $q^2 = \frac{3}{4}(u'^2+w'^2)$, assuming that $v'^2 \approx \frac{1}{2}(u'^2+w'^2)$, as is common
practice in laboratory flat plate boundary layer data (Hinze 1975). $-u'w'$ gradually increases from less than 0.5 cm$^2$/s$^2$ near the bottom to a nearly uniform value of about 1 cm$^2$/s$^2$. The values of $-u'w'$ are significantly lower than $q^2$, which is also smallest near the bottom, and has a general trend of increasing with elevation. The ratios between $-u'w'$ and $q^2$ obtained for the present data (Table B1), exhibit similar trends to the ratio between the shear stress and the turbulent kinetic energy (Townsend’s structure parameter) reported in the literature. The value is the lowest (0.07) near the bottom, and fluctuates between 0.10 and 0.15 at the higher elevations. According to Hinze (1975), which includes numerous references, the value of $-u'w'/q^2$ in most of the low Reynolds number, flat plate boundary layers is about 0.16, vanishing at both edges. In the present data, the ratio in most of the measurement stations is slightly lower. The marked decrease of $-u'w'/q^2$ at the bottom is consistent with $z/\delta<0.05$ in the laboratory flows. The lower ratio and its decrease near the bottom in the present data are consistent with the values presented by Townsend (1976) for a boundary layer with a positive pressure gradient: 0.13 in the outer layer and 0.095 in the inner layer for small pressure gradients (where the exponent of the power law describing the variation of free stream velocity is $-0.15$), and 0.14 in the outer layer and 0.07 in the inner layer for large pressure gradients (where the exponent of the power law is $-0.255$). The present data is also consistent with the value of 0.13 reported by Townsend for the outer layer of a boundary layer with no pressure gradient, but the reduction in the inner layer in that case is very slight (to 0.12), i.e., considerably less significant than in the present data. Saddoughi and Veeravalli (1994) measured values slightly smaller than 0.13, reducing to less than 0.10 near the wall and the outer edge, in a high Reynolds number boundary layer. The data of Sanford and Lien (1999), from a tidal bottom boundary layer, shows a similar trend, with values of $-u'w'/q^2=0.12$ at $z=0.6$ m and $-u'w'/q^2=0.13$ at $z=1.4$ m. Despite contamination of the
turbulence data by the wave induced unsteady flow, the presence of external pressure gradients (or Coriolis forcing), and the non-flat bottom, there is clear agreement, in both characteristic values and trends, between our data and results reported in the literature.

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Figure 3: Sample vector map at the lowest measurement station (the center is 20 cm above the bottom), constructed from two 1024 x 1024 pixel images taken 20 ms apart. The average horizontal velocity for this map is 4.83 cm/s at $z = 11.3$ cm, and 12.03 cm/s at $z = 28.7$ cm.

Figure 4: Vertical distributions of mean horizontal ($U$) and vertical ($W$) velocities, calculated from 130 vector maps recorded at 1 Hz, at each of the six elevations. Time interval between measurement stations – 15 minutes.

Figure 5: Vertical distributions of mean horizontal ($U$) velocity, calculated for 5 subsets of 26 vector maps each (each point is the average of 754 vectors), and from the entire series of 130 vector maps recorded at 1 Hz, at each of the six elevations. The center of the sample area is at: (a) $z = 128$ cm, (b) $z = 106$ cm, (c) $z = 82$ cm, (d) $z = 62$ cm, (e) $z = 44$ cm, (f) $z = 20$ cm.

Figure 6: Time evolution of the mean velocity at different elevations. Each point on the figure is the average of data from a 1.3 cm “strip” centered at the given elevation (i.e., the average of $3 \times 29 = 87$ measured data points).
Figure 7: Velocity fluctuation (((u(x,z,t)-U(z)), (w(x,z,t)-W(z)))) vector maps; u(x,z,t) and w(x,z,t) are velocity components of a vector; U(z) and W(z) are averages of u, w at constant z, over the entire data set. (a)-(d): Sequence of 4 maps, measured at 1 s intervals, at the lowest station. Note the structures designated “1” and “2” being convected across the sample area. (e), (f): Sample individual maps, at the highest measurement station. Note small structures embedded within larger ones.

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Figure 9: Running RMS averages of velocity fluctuations, \( \sqrt{\frac{1}{N} \sum_N (u'(z)^2) } \), \( \sqrt{\frac{1}{N} \sum_N (w'(z)^2) } \), and running average of \( u'(z)w'(z) \), \( \frac{1}{N} \sum_N (u'(z)w'(z)) \), as a function of number of points, N, used to compute the average. (a) \( z = 128 \) cm, (b) \( z = 12 \) cm.

Figure 10: Vertical profiles of RMS velocity fluctuations (Eqs. 3.7, 3.8). \( U(z) \) is the average horizontal velocity, calculated from the entire 130 vector maps, at each elevation. \( U_{max} \) is the largest value of \( U(z) \).

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Figure 12: Sample of velocity interpolation from original data onto a regularly spaced grid. The interpolated $u$ at $x = 25.1$ cm is the weighted average of 6 original data points from three consecutive vector maps, lying between $x = 24.5$ cm and $x = 25.8$ cm. The interpolated $u$ at $x = 24.5$ cm is the weighted average of 5 original data points (two from $t = 1$ s, two from $t = 2$ s, and one $t = 3$ s), lying between $x = 23.9$ cm and $x = 25.1$ cm.

Figure 13: Sample spectra of turbulent velocity fluctuations, calculated from expanded vector map. Solid lines: $E_{11}$; dashed lines: $E_{33}$; dotted lines: Nasmyth universal spectrum (which has a $(-5/3)$ slope in the inertial range).

Figure 14: Sample spectra of turbulent kinetic energy dissipation, calculated from the extended vector maps. The logarithmic plots are shown to emphasize the spectral peaks. Solid lines: $k_1^2E_{33}$; dashed lines: $k_1^2E_{11}$.

Figure 15: Dissipation spectra, calculated from the fluctuations of the vertical velocity ($k_1^2E_{33}$). Solid lines: present data; dashed lines: Nasmyth universal spectrum.

Figure 16: Estimates of dissipation rate at the 6 measurement stations, obtained using different estimation methods. Note, that this figure represents both spatial and temporal variations due to the 15 min time delay between successive stations.

Figure 17: Sample velocity spectra obtained from averaging spatial distributions of individual vector maps, and from the extended series. Solid lines: $E_{11}$ from individual maps; dotted lines: $E_{33}$ from individual maps; solid symbols: $E_{11}$ from extended series; open symbols: $E_{33}$ from extended series.
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Figure 19: Sample streamwise and spanwise velocity spectra obtained from averaging spatial distributions of individual vector maps. Solid lines: $E_{11}(k_1)$; dashed lines: $E_{11}(k_3)$; dashed-dotted lines: $E_{33}(k_1)$; dotted lines: $E_{33}(k_3)$. $k_1$, $k_3$: streamwise and spanwise wavenumbers, respectively.

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Table 2: Results of least-squares best fit to (-5/3) slopes at intermediate wavelength for the sample spectra in Figure 13.

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Table 4: Energy content in the range of wavenumbers where the individual vector maps (IM) and the extended vector map (ES) overlap.

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<thead>
<tr>
<th>Elevation (cm)</th>
<th>$A_{11} \times 10^4$</th>
<th>$A_{33} \times 10^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>2.2</td>
<td>2.3</td>
</tr>
<tr>
<td>106</td>
<td>3.5</td>
<td>3.0</td>
</tr>
<tr>
<td>82</td>
<td>2.4</td>
<td>2.1</td>
</tr>
<tr>
<td>62</td>
<td>2.5</td>
<td>2.6</td>
</tr>
<tr>
<td>44</td>
<td>1.8</td>
<td>1.7</td>
</tr>
<tr>
<td>12</td>
<td>2.3</td>
<td>1.9</td>
</tr>
</tbody>
</table>
Table 3: Estimates of kinetic energy dissipation for the sample data of Figures 13 and 14.

<table>
<thead>
<tr>
<th>Elevation (cm)</th>
<th>“Direct”</th>
<th>$\varepsilon_D \times 10^6$ (m$^2$/s$^3$)</th>
<th>$\eta$ (mm)</th>
<th>$\varepsilon_{LF} \times 10^6$ (m$^2$/s$^3$)</th>
<th>$\varepsilon_{DS} \times 10^6$ (m$^2$/s$^3$)</th>
<th>$\varepsilon_{AS} \times 10^6$ (m$^2$/s$^3$)</th>
<th>$\varepsilon_{SG} \times 10^6$ (m$^2$/s$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>128</td>
<td>3.9</td>
<td>5.8</td>
<td>0.92</td>
<td>4.3</td>
<td>3.7</td>
<td>4.8</td>
<td></td>
</tr>
<tr>
<td>106</td>
<td>6.0</td>
<td>8.8</td>
<td>0.82</td>
<td>8.2</td>
<td>5.3</td>
<td>10.5</td>
<td></td>
</tr>
<tr>
<td>82</td>
<td>3.3</td>
<td>5.2</td>
<td>0.95</td>
<td>4.1</td>
<td>3.0</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>4.1</td>
<td>7.3</td>
<td>0.90</td>
<td>5.7</td>
<td>3.7</td>
<td>5.8</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td>3.1</td>
<td>3.7</td>
<td>0.97</td>
<td>3.2</td>
<td>2.8</td>
<td>2.8</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>4.1</td>
<td>4.5</td>
<td>0.90</td>
<td>4.1</td>
<td>3.9</td>
<td>7.9</td>
<td></td>
</tr>
</tbody>
</table>
Table 4: Energy content in the range of wavenumbers where the individual vector maps (IM) and the extended vector map (ES) overlap.

<table>
<thead>
<tr>
<th>Elevation (cm)</th>
<th>Average of individual vector map spectra (IM)</th>
<th>Extended series (ES)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$e_1 \times 10^7$ (m²/s²)</td>
<td>$e_3 \times 10^5$ (m²/s²)</td>
</tr>
<tr>
<td>128</td>
<td>3.6</td>
<td>3.7</td>
</tr>
<tr>
<td>106</td>
<td>4.4</td>
<td>4.3</td>
</tr>
<tr>
<td>82</td>
<td>2.8</td>
<td>3.0</td>
</tr>
<tr>
<td>62</td>
<td>3.6</td>
<td>4.4</td>
</tr>
<tr>
<td>44</td>
<td>3.1</td>
<td>3.0</td>
</tr>
<tr>
<td>12</td>
<td>4.6</td>
<td>4.5</td>
</tr>
</tbody>
</table>
Table 5: Dissipation estimates obtained from the spectra of the individual vector maps (IM) and the spectra of the extended vector map (ES).

<table>
<thead>
<tr>
<th>Elevation (cm)</th>
<th>Average of individual vector map spectra (IM)</th>
<th>Extended series (ES)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon_{LF} \times 10^6$ (m$^2$/s$^3$)</td>
<td>$\varepsilon_{DS} \times 10^6$ (m$^2$/s$^3$)</td>
</tr>
<tr>
<td>128</td>
<td>5.6</td>
<td>6.1</td>
</tr>
<tr>
<td>106</td>
<td>10.7</td>
<td>10.9</td>
</tr>
<tr>
<td>82</td>
<td>4.5</td>
<td>5.3</td>
</tr>
<tr>
<td>62</td>
<td>6.1</td>
<td>6.6</td>
</tr>
<tr>
<td>44</td>
<td>4.4</td>
<td>5.3</td>
</tr>
<tr>
<td>12</td>
<td>6.2</td>
<td>9.1</td>
</tr>
</tbody>
</table>
Table B1: Estimates of Townsend’s structure parameter at the six measurement stations.

<table>
<thead>
<tr>
<th>Elevation (cm)</th>
<th>(-u'w'/q^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>118-138</td>
<td>0.12</td>
</tr>
<tr>
<td>96-116</td>
<td>0.13</td>
</tr>
<tr>
<td>72-92</td>
<td>0.15</td>
</tr>
<tr>
<td>52-72</td>
<td>0.10</td>
</tr>
<tr>
<td>34-54</td>
<td>0.13</td>
</tr>
<tr>
<td>10-30</td>
<td>0.07</td>
</tr>
</tbody>
</table>