APPLICATION OF HPIV DATA OF TURBULENT DUCT FLOW FOR TURBULENCE MODELING

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ABSTRACT

Holographic PIV is used to measure the three-dimensional velocity distribution of fully developed turbulent flow within a square duct at $Re_H = 1.2 \times 10^5$. The data is used for mapping the structure of the filtered, three-dimensional vorticity, strain-rate and subgrid-scale (SGS) stress tensor distributions. Visualizations of iso-vorticity surfaces show structures that are only slightly elongated, as opposed to the long and thin "worms" observed in DNS for unfiltered turbulence. The structure of these fields is studied further by measuring absolute and relative orientations of characteristic vectors. The fluctuating vorticity has a preferred direction aligned at about $45^\circ$ to the mean flow. The preferential alignment of the vorticity with the intermediate eigenvector of the strain-rate tensor observed previously in DNS data is also found in this high Reynolds number shear flow. The PDF of the intermediate eigenvalue shows some differences compared to DNS results. Two of the SGS stress eigenvectors are nearly randomly aligned compared to the most extensive and intermediate eigenvectors of the filtered strain-rate tensor. In contrast, the most extensive stress eigenvector has a significant correlation with the direction of the most compressive strain-rate eigendirection. These results help to shed light on the relative success of eddy viscosity models in predicting the SGS dissipation, but their failure in predicting the actual stresses. In addition, the observed preferential alignments of the filtered vorticity with two stress eigendirections may point to new modeling approaches based on vorticity.

1. INTRODUCTION

Direct Numerical Simulation (DNS) has provided valuable information about turbulent flow phenomena and played an important role in the development of improved physical models (Moin & Mahesh, 1998). A limitation of DNS is that all scales of the flow must be properly resolved. Therefore, the technique is limited to low Reynolds number flows. Experimental data is required if flows at high Reynolds numbers are to be studied.

Several successful attempts have been made in recent years to measure the three-dimensional velocity distribution in a finite sample volume using Holographic Particle Image Velocimetry (HPIV). Barnhart et al (1994) developed a phase-conjugate holographic system to measure the air flow in a turbulent pipe. Meng and Hussain (1995) used an in-line recording, off-axis viewing setup to measure an instantaneous flow field in a vortex ring. Zhang et al (1997) introduced a hybrid HPIV system and measured a 3-D velocity field of turbulent water flow in a square duct.

In order to improve turbulence modeling for Large Eddy Simulation (LES), in recent years we have performed tests of subgrid-scale (SGS) stress models based on 2-D PIV data (Liu et al, 1994, 1995, 1999, and Meneveau and Katz, 1999). These measurements inherently provided only partial data on the stress and strain distributions. At present, we do not have experimental data on the three-dimensional velocity distribution of a high Reynolds number turbulent flow. Consequently, we do not have information on the full orientation of the SGS stresses relative to the filtered strain rate tensor or vorticity vector. Such data are critical for continued development of physics based SGS stress models. An effort to obtain such data using HPIV started several years ago (Zhang et al, 1997), as summarized in section 2. The present paper focuses on use of this technology to study trends of the flow structure by comparing the alignment of the filtered vorticity vector, filtered strain-rate eigendirections and eigendirections of the SGS stress tensor. Considerable attention has already been given to determining the geometrical relationship between the vorticity
vector and the eigenvectors of the strain-rate tensor. Ashur \textit{et al.} (1987) and Vincent and Meneguzzi (1994) used DNS, and Tsinober \textit{et al.} (1992) used hot-wire measurements for this purpose. Several important observations were made, such as (a) regions of high vorticity magnitude are of tubular shape ("worms"), (b) the vorticity vector shows preferential alignment with the intermediate eigenvector of the strain-rate tensor, and (c) the probability density function (PDF) of the intermediate strain-rate eigenvalue shows a preponderance of axisymmetric motions. The latter conclusion was based on the properly redefined scaling by Lund and Rogers (1994). By necessity, these conclusions were reached based only on low Reynolds number data. In the present work, we determine whether similar conclusions hold for high Reynolds number turbulence at intermediate scales in the inertial range of turbulence. Issues of vector and tensor alignment are extended to the context of SGS modeling.

2. EXPERIMENTAL SETUP

The current velocity measurements are performed in a water flow within a vertical, square duct with a width \( H = 57 \text{ mm} \). The test section is located \( 36H \) downstream of the duct inlet, where conditions close to fully developed turbulent flow are established. The walls of the test section are made of flush mounted glass windows, creating an unobstructed sample volume of \( 57 \text{ mm} \times 57 \text{ mm} \times 45 \text{ mm} \) (the smaller dimension is in the axial direction). The present data sets are obtained at a center-line mean velocity \( \bar{u}_{\text{max}} = 2.1 \text{ m/s} \). The corresponding Reynolds number based on \( \bar{u}_{\text{max}} \) and the duct width is \( Re_u = 1.2 \times 10^5 \). More details about this setup can be found in Zhang \textit{et al.} (1997).

3. DATA PROCESSING

Measurements of the instantaneous three-dimensional velocity distribution within the sample volume are performed using Holographic Particle Image Velocimetry (HPIV). A schematic description of the optical setup is presented in Fig. 1 (for details, see Zhang \textit{et al.}, 1997). In order to maintain high accuracy (1 - 2%) in obtaining all three velocity components, we simultaneously record two perpendicular, off-axis, double exposure holograms of a flow field seeded with 20 \( \mu \text{m} \) diameter, neutrally buoyant particles. Each hologram is used to determine the two velocity components that are normal to its optical axis. The redundant component helps to match the two data sets precisely. The holographic film (70 mm, AGFA 10E75) is developed and mounted on a reconstruction system. The three-dimensional image is scanned with a video camera equipped with a microscope objective, plane by plane at a step of 0.33 mm (corresponding to the vector spacing within each plane) between adjacent planes. The images are then stored on disk for later batch data processing.

![Fig.1 Schematic of the optical setup for recording two perpendicular holograms.](image)

In this paper we present velocity measurements of the turbulent flow away from the immediate vicinity (5.25 mm) of the walls. Thus, the current sample volume is the central 46.5 mm \( \times \) 46.5 mm \( \times \) 44.5 mm. The image in each digitized plane is analyzed using standard PIV procedures, i.e. the image is divided into 192 pixel \( \times \) 192 pixel (0.93 mm \( \times \) 0.93 mm) interrogation windows and the mean displacement is determined from the auto-correlation function of the intensity distribution. Due to improved particle seeding and signal-to-noise ratio (achieved mostly by "fine tuning" of the procedures) the present yield of correct vectors in each hologram is above 90%, significantly higher than the 80% production rate reported in our previous work (Zhang \textit{et al.}, 1997). With 65% overlap between windows, the resulting vector spacing is 0.33 mm and the entire sample volume contains 136 \( \times \) 130 \( \times \) 128 velocity vectors. We use an improved procedure for determining the velocity (Tao \textit{et al.}, 1997). It is based on compressing an enhanced image and then computing the correlation directly from the compressed image, similar to Hart (1998). The enhancement involves histogram equalization and elimination of low gray levels to remove background noise (Bertuccioli \textit{et al.}, 1997, Roth and Katz, 1999). The compressed data maintains the gray level and location of pixels with level above the threshold value. The typical compression ratio is about 20:1 to ensure an accuracy comparable to that of Roth and Katz (1999). Data evaluation and error checking are similar to Roth \textit{et al.} (1995) and Sridhar and Katz (1995). This approach enables us to calculate particle displacements at higher magnification without having to analyze an increasingly larger database. In fact, working with 192\( \times \)192 pixel windows causes only a marginal increase in the analysis time compared to typical 64 pixel \( \times \) 64 pixel windows.

4. RESULTS AND DISCUSSION
In the present paper we present results obtained by processing two instantaneous 3-D vector maps. Their quality, determined by evaluating how well the data satisfy the continuity equation, is comparable to our previous data set (Zhang et al, 1997). We will return to this issue while discussing the distribution of strain rates. In the current two data sets, using estimating procedures discussed in Liu et al (1994), we obtain a Kolmogorov scale \( \eta \approx 100 \) mm, a Taylor microscale of \( \lambda = 3.4 \) mm, and consequently, \( Re_l = 310 \).

The data are spatially filtered at a scale \( \Delta \), and we calculate the SGS stresses (all six components), filtered strain rate tensor and the 3-D vorticity distribution. We then use the data to observe trends in the alignment of the vorticity relative to the mean flow and the relative orientations of the filtered vorticity vector, of eigenvectors of the filtered strain-rate tensor and of the SGS stress tensor.

Spatial filtering of the velocity field is performed according to (an overhead tilde denotes the filter operation hereafter):

\[
\tilde{u}_i(x) = \int \int \int u(x') F_\Delta(x-x')d'x',
\]

where \( F_\Delta(x) = \Delta^{-3} \) at \( |x| < \Delta/2 \) and zero outside of this domain, is a spatial low-pass box filter with size \( \Delta = 3.3 \) mm, corresponding to 10 vector spacings. The filtered vorticity, \( \tilde{\omega}_i \), filtered strain-rate tensor \( \tilde{S}_ij \), and sub-grid scale stress tensor \( \tau_{ij} \), defined as:

\[
\tilde{\omega}_i = \varepsilon_{ij} \partial \tilde{u}_j / \partial x_i, \tag{2}
\]

\[
\tilde{S}_{ij} = (\partial \tilde{u}_i / \partial x_j + \partial \tilde{u}_j / \partial x_i)/2, \tag{3}
\]

\[
\tau_{ij} = \tilde{u}_i \tilde{u}_j - \tilde{u} \tilde{u}, \tag{4}
\]

are all calculated directly from the data.

**Topology of Regions of High Vorticity:** Figure 2 shows a surface plot of iso-vorticity magnitude at a threshold of \( \omega = 4 \sigma_{\omega} \), where \( \sigma_{\omega} \) is the RMS value of the filtered vorticity magnitude. For this case \( \sigma_{\omega} = 25.2 \) s\(^{-1}\), i.e. \( \sigma_{\omega} = 0.67 \bar{\omega}_{\max} / H \). For the second data set (not shown) \( \sigma_{\omega} = 24.4 \) s\(^{-1}\), essentially the same value. The mean flow is along the \( y \) direction. As is evident from this plot the regions of high filtered vorticity magnitude form structures with only weak elongation, in contrast to those from DNS analyses (Ashurst et al, 1987; Vincent and Meneguzzi, 1994) that look significantly thinner and more “worm”-like. Note that the use of other vorticity levels for the surface plots leads to the same conclusion (data not shown). The contours of higher vorticity are located within the shown contours and appear as smaller blobs, whereas the contours of lower vorticity appear as even fatter blobs.

**Orientation of the Vorticity Vectors:** Next, we examine the alignment of the vorticity vectors relative to the streamwise (\( y \)) direction. The probability distributions (PDF) of the alignment angle, \( \theta \), of the total vorticity vector relative to the streamwise direction are shown in Figure 3, for both data sets. The PDFs are normalized by \( \sin \theta \) so that totally random angles would correspond to a uniform PDF. In order to take into account only regions of high vorticity shown in Fig. 2, the PDF is formed only from vectors that have a magnitude between \( 3.9 \sigma_{\omega} \) and \( 4.1 \sigma_{\omega} \). We use 16 bins, 11.25\(^\circ\) each, within the 180\(^\circ\) angle range. Angles between 0\(^\circ\) to 90\(^\circ\) indicate that the axial component of the vorticity is positive, whereas angles between
90° and 180° indicate a negative axial component. It is evident that most of the vorticity vectors are not aligned with the mean flow. Instead, they are more or less aligned normal to it. Recall, however, that the mean vorticity in this flow lies in the x-z plane. If the mean vorticity, approximated here for convenience as the axial (y-axis) average of all \( \vec{\omega}_i(x,y,z) \) and \( \vec{\omega}_j(x,y,z) \) components, is subtracted from the vorticity field, the resulting PDF has a significantly different distribution, as demonstrated in Fig. 3 (circles). Again, we only present here the alignment of vorticity vectors with magnitude between 3.9\( \sigma_v \) and 4.1\( \sigma_v \). It is clear that the preferred alignment of the vorticity is inclined by about 45° to the streamwise direction. The symmetric peaks indicate that the same conclusion applies to vorticity with positive and negative axial components. When the analysis is repeated with lower values of vorticity (results not shown), the distribution becomes increasingly more uniform. A remnant of the peaks can be still detected with values around 2\( \sigma_v \). Essentially disappears when the range is around 1\( \sigma_v \).

To determine the most preferred mode of straining, Lund and Rogers (1994) consider the PDF of the following quantity:

\[
s^* = \frac{-3\sqrt{6}abc\gamma}{(a^2 + b^2 + c^2)^{3/2}}.
\]  

They show that \( s^* \) yields physically more meaningful results than a parameter previously defined by Ashurst et al (1987) since a random velocity field gives a uniform PDF of \( s^* \) but a non-uniform one in the case of Ashurst et al's parameter. From DNS, Lund and Rogers (1994) find that the most probable state is at \( s^* = 1 \), indicating that the most likely flow configuration is axisymmetric expansion (\( \beta = \gamma = -0.5\alpha \)). We also compute the distribution of \( s^* \) using 30 bins across all the possible values, and the results are presented in Figure 5. Note that \( P(s^*) \) is not zero beyond \( \pm 1 \), as should be the case for perfectly divergence-free data (Lund and Rogers, 1994). The reasonably small fraction of values beyond the allowable limits is consistent with the trends of Fig. 4. The present data indicates that the peak probability occurs at \( s^* = 0.55 \). Thus, although axisymmetric expansion is quite likely, it is not the most probable state in this flow, for the filtered velocity field.

**Alignment of Vorticity and Strain-rate Tensor:** Regarding the relative alignment of the vorticity vector and strain-rate eigenvectors, from DNS it has already been shown that the vorticity vector is preferentially aligned with the eigenvector corresponding to the intermediate eigenvalue of the strain-rate tensor (i.e. \( \beta \)). Using the present data, Fig.6 shows the PDF of \( \cos\theta \), where \( \theta \) is the angle between the filtered vorticity vector and the eigenvectors of the filtered strain rate tensor

\[\begin{align*}
\kappa &= \frac{(\alpha + \beta + \gamma)^2}{(\alpha^2 + \beta^2 + \gamma^2)}.
\end{align*}\]

Fig.4 Cumulative probability of \( \kappa \), defined in eq. (5).

Ideally \( \kappa \) should be zero at every point. Figure 4 shows the cumulative probability of \( \kappa \). As is evident from the results, 62% of the points have a square-error coefficient less than 0.05 and 80% of the points have errors less than 0.2. These results, whose values are computed from the sum of three velocity gradients, are consistent with our claimed velocity measurement accuracy of 1 - 2% (Zhang et al, 1997).
3-D experimental data. For the intermediate strain rate eigendirection, β, there is a substantially increased probability towards \( \cos \theta = 1 \). There is little correlation between the vorticity and the alignment of the largest extensive strain, \( \alpha_s \), and the vorticity is least likely to point in the most compressive direction (along that of \( \gamma \)).

**Alignments with the SGS Stress Tensor:** The probabilities of alignment of the negative SGS stress tensors with the filtered strain-rate tensors and the filtered vorticity vector are plotted in Fig.’s 7 and 8, respectively. In both cases we only consider the deviatoric part of the SGS stress i.e.:

\[
\tau'_{ij} = \tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij}.
\]

The eigenvalues of the negative SGS stress are organized such that \( \alpha_s \) is a unit vector aligned with the eigenvector of the most compressive SGS stress and \( \gamma_s \) is aligned with the most extensive stress. As is evident from Fig. 7, there is no significant correlation between the direction of \( \alpha_s \) and \( \alpha_t \) as well as between \( \beta_s \) and \( \beta_t \). However, the most compressive strain-rate component, \( \gamma_s \), has an increased correlation with the most positive (extensive) SGS stress component. This increased probability peaks at \( \cos \theta = 0.8 \), corresponding to an angle of 37°. These trends may explain the reason that eddy viscosity models, where the SGS stress is assumed to be aligned with the (negative) filtered strain rate, cannot reproduce the correct dynamics of the SGS stresses, but they can reproduce the correct trends in SGS dissipation (see e.g. Liu et al., 1994). A strong correlation between the largest extensive stress and the most compressive strain would result in SGS energy dissipation \( (\Pi^1 = -\tau_{kk} \delta_{ij}) \) with the correct sign (positive), with the other components contributing significantly less to the overall dissipation due to their random alignment.

On the other hand, in Fig. 8, we observe considerable alignment between the filtered vorticity vector and the most negative (compressive) SGS stress component. Moreover, it appears that there is also an increased alignment between the vorticity vector and the intermediate SGS stress component. Finally, the most probable alignment of the most extensive stress eigenvector appears to be perpendicular to the filtered vorticity vector. These trends provide new insights into the proper geometrical relationship among the filtered vorticity vector, the filtered strain-rate tensor and the SGS stress tensor.
SUMMARY

Holographic PIV measurements of a fully developed turbulent duct flow at Re₈ = 1.2 × 10⁵ are used to study the structure of the three-dimensional vorticity field, and the geometrical relationship between the filtered vorticity vector, the resolved strain-rate tensor and the subgrid-scale stress tensor. The PDFs of the streamwise alignment angle of peak values of fluctuating vorticity show a preferred direction, aligned at about 45° to the mean flow. Filtered vorticity isosurfaces display the topology of slightly elongated “blobs” as opposed to thin tubes observed in DNS of unfiltered turbulence. While the PDF of the strain-rate eigenvalues shows a general tendency towards the limit of axisymmetric expansion, as opposed to previous DNS results the most likely value does not correspond to axisymmetric expansion. In good agreement with DNS for unfiltered data, the filtered vorticity shows preferential alignment with the intermediate eigenvector of the filtered strain-rate tensor. Two of the SGS stress eigenvectors are nearly randomly aligned compared to the most extensive and intermediate eigenvectors of the strain-rate tensor. It is somewhat surprising that the most expansive eigendirection of the strain-rate (vortex stretching) is not met with any preferential "resistance" by the SGS stress tensor. In contrast, the most extensive stress eigenvector has a significant correlation with the direction of the most compressive strain-rate eigendirection, i.e. the SGS stress tensor preferentially "resists" the compression (i.e. reduction of length-scale) of fluid elements. These results may help to shed light on the relative success of eddy viscosity models in predicting the SGS dissipation, but their failure in predicting the actual stresses. Finally, preferential alignment between the filtered vorticity with two stress eigendirections is observed. Implications of these results for SGS modeling will be explored in future work.

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